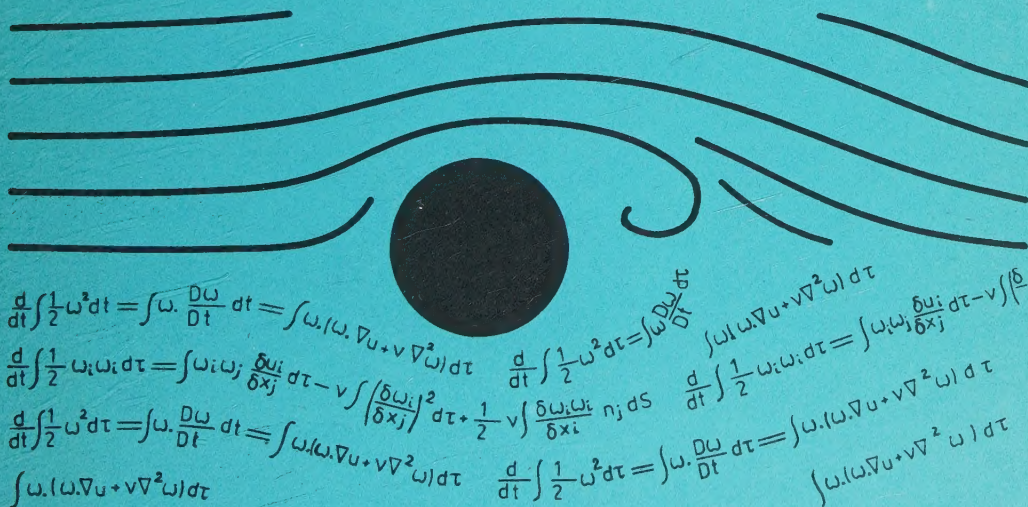


STUDIES ON THE NATURE OF LOCAL SCOUR

by

Peder Hjorth

1975





STUDIES ON THE NATURE OF LOCAL SCOUR

av

Peder Hjorth, civilingenjör, Mlm

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LIST OF CONTENTS

	Page
LIST OF CONTENTS	I
ABSTRACT	III
LIST OF SYMBOLS	V
1 INTRODUCTION	1
2 PREVIOUS INVESTIGATIONS	3
2.1 Scour around bridge piers	3
2.2 Scour around pipes	25
2.3 Initiation of motion and sediment transport	31
2.4 Effects of pressure gradients and seepage forces	40
2.5 Present state of affairs	47
2.5.1 Location of maximum scour and form of scour hole	49
2.5.2 Influence of shape	49
2.5.3 Influence of angle of attack	49
2.5.4 Spacing of piers	49
2.5.5 Influence of approach velocity	50
2.5.6 Pier width	53
2.5.7 Sediment properties	53
2.5.8 Influence of water depth	54
2.5.9 Lift and seepage forces	54
2.5.10 Sediment transport formulas	55
2.5.11 Scour protection	55
2.5.12 Pipeline scour	56
3 PHILOSOPHY OF APPROACH	57
4 ANALYSIS OF THE PROBLEM	60
4.1 Two-dimensional flow over pipeline	60
4.2 Three-dimensional flow about a cylinder	62
5 ANALYTIC DESCRIPTIONS OF THE FLOWS	67
5.1 Introductory remarks	67
5.2 The pipeline case	73
5.3 Flow around cylinders	81
5.3.1 The secondary flow upstream of a cylinder	82
5.4 A three-dimensional model	90
5.4.1 Flow around circular pier	96
5.4.2 Flow around rectangular pier	99
5.4.3 Flow around sharpnosed pier	103
5.5 Turbulent effects	112
6 EXPERIMENTAL INVESTIGATION	119
6.1 Flow pattern and velocity distribution	119
6.1.1 Cylindrical piers	119
6.1.2 Rectangular cylinders	124
6.1.3 Devices to weaken the horse-shoe vortex	129
6.1.4 Effect of turbulence level of the approach flow	129
6.1.5 Flow over pipelines	130
6.2 Boundary shear stresses around cylinders and pipe-lines	132

	Page
6.2.1 Circular cylinders	132
6.2.2 Protecting collar	138
6.2.3 Rectangular piers	139
6.2.4 Pipelines	141
6.3 Pressure distribution about cylinders and pipes	143
6.3.1 Circular cylinders	145
6.3.2 Rectangular cylinders	150
6.3.3 Rectangular pier turned 45°	154
6.3.4 Pipelines	156
6.4 Statistical analysis of pressure fluctuations	159
6.4.1 Probability density function	159
6.4.2 Auto correlation function	160
6.4.3 Cross correlation functions	161
6.5 Sediment movement	162
6.6 Discussion of the experimental results	169
7 CONCLUSIONS	171
7.1 Scour around pipelines	171
7.2 Scour around bridge piers	172
8 EXAMPLE OF HOW TO USE THE RESULTS OF THIS REPORT TO DESIGN A SCOUR PROTECTION	175
LIST OF REFERENCES	180

ABSTRACT

This study is mainly concentrated to the mechanism of the local scour process associated with pipelines and cylindrical bridge piers. The pipelines are supposed to be lying on or to be partly embedded in a uniform sediment bed. The oncoming flow is supposed to be at right angles to the axis of the pipeline.

The cylindrical bridge piers investigated are of three shapes, circular, square and square turned 45° to have an edge facing the oncoming flow.

The study is restricted to the case of flow over a unscoured bed, that is, the results are most useful for design of scour protection devices or as a starting point for estimates of scour development.

The experimental techniques employed are; Hot-film anemometry to establish the flow properties within the fluid region and to determine the magnitude of the boundary shear near the obstacle. A plaster of Paris model technique to establish the friction lines (lines whose tangents indicate the direction of shear) on the bed and on the surface of the obstacle.

Pressure measurements at the bed to determine the temporal and spatial pressure distributions.

A theoretical analysis involving vorticity considerations is presented. The theoretical results are in good agreement with the experimental results of this and other studies.

The results of the study may be summarized as follows; Scour under pipelines is caused by pressure gradients in the bed set up by the pressure difference between the stagnation zone upstream and the wake region downstream. Thus, the greater the length of seepage between these regions the safer the pipe.

For the cylinders it is shown that each shape creates a specific boundary shear distribution, the strength of which is directly proportional to the upstream boundary shear for moderate to high Reynolds numbers. Theoretically it is shown that the boundary shear distribution is weakly dependent on the ratios pier width to boundary roughness and depth of flow to boundary roughness.

The observed distributions of boundary shear are contrary to expectations in the respect that the regions of maximum boundary shear do not coincide with the regions of maximum secondary flow. Thus, the square cylinder shape creates less boundary shear than the circular. By the same token, maximum boundary shear is not found near the stagnation plane but at some oblique position.

Studies of the pressure distributions show that they are similar to those of two-dimensional flows. Consideration of the resulting seepage forces show that these should absolutely be included in the stability analysis of the foundation.

The secondary flows generated by the cylinders are found to be relatively weak. However, they are strong enough to reverse the flow in the lower layers in a region upstream of the cylinder. The sediment supply to the region near the cylinder and to the wake region is thereby effectively blocked. This leads to a sediment deficit in these regions which has to be compensated for by scour. This mechanism is used to explain why a scour protecting layer is more effective when placed below bed level.

Finally an example is given of how to use the findings of this study to design a rip-rap scour arresting mat.

LIST OF SYMBOLS

A	rate of shear
a	half-width of obstacle
B	width of obstacle
B_w	function describing sediment properties
C	dimensionless constant
	auxiliary function
C_D	drag coefficient
C_p	dimensionless pressure coefficient
c	coefficient
D	depth at which the vertical seepage velocity is zero
D_L	Lacey regime depth
D_s	depth of scour below water level
d	sediment size
d_{35}	sediment size for which 35 % by weight is finer
d_p	equilibrium scour depth below bottom of pipeline
d_s	depth of scour below bed level
ds	length of curve element
e	voids ratio
F_s	seepage force per unit volume
f	silt factor
	frequency
g	acceleration of gravity
H	depth of flow
H_p	depth of stone protection below water level
h	distance between adjacent skin-friction lines
K_1	factor accounting for effect of pier shape
K_2	factor accounting for effect of angle of attack
K_p, K_R	complex constant of the Schwartz-Chistoffel transform
k	permeability index
k_s	equivalent sand roughness
k_x, k_y	permeability index of the x- and y- direction respectively
L	characteristic length
L_p, L_R	complex constant of integration
l	length parameter
m	mean size of sediment in mm

N_s	sediment number $U/(s_s - 1)^{1/2}$
N_{sc}	sediment number at incipient motion
n	porosity index
n_1	coefficient
P	wetted perimeter
P_b	average bearing pressure
p	pressure
P_w	pressure at the wall
Q	auxiliary function
q	discharge per unit width
q_{lim}	limiting discharge per unit width
R	hydraulic radius
Re	Reynolds number Ul/ν
Re^*	shear velocity Reynolds number $u_* d/\nu$
r	packing factor
	position vector relative to fluid element
	cylinder coordinate
S	energy slope
Sh	Strouhal number fB/U
s_s	relative density of solid
T	wave period
t	time or 'drift'
t_1	auxiliary drift-function
U	velocity of undisturbed flow
U_∞	velocity outside vorticity region
U_c	critical mean velocity
U_p	mean velocity above crest of pipe
u	velocity component in the x-direction
u_0	a characteristic velocity
u_*	friction velocity $(\tau/\rho)^{1/2}$
u_r, u_θ	
u_z	velocity components in the r- θ -z directions respectively
V	velocity field
V_0	velocity field
V_1	velocity field
V_f	volume of vorticity field
V_s	volume of sediment particle

v	velocity component in the y-direction
W	weight
w	velocity component in the z-direction particle fall velocity
x	cartesian coordinate
y	cartesian coordinate elevation
z	vertical coordinate point in the complex z-plane, $x + iy$
α	exponent apparent angle of repose
α_c	critical bed slope
γ	specific weight of fluid
δ	height of boundary layer
ϵ	volume flow
ζ	point in the complex ζ -plane $\xi + i\eta$
η	coordinate in the complex ζ -plane
θ	cylinder coordinate
θ'	dimensionless effective boundary shear
κ	curvature of surface
μ	dynamic viscosity
ν	kinematic viscosity
ξ	coordinate of the complex ζ -plane
ρ	fluid density radius of curvature
ρ_s	solid density
σ	standard deviation
τ_c	critical boundary shear
τ_0	boundary shear
τ_0'	effective boundary shear
τ_0''	boundary shear associated with form resistance
τ_0^*	dimensionless boundary shear
ϕ	angle of repose sediment transport parameter
Φ	velocity potential
ψ	sediment transport parameter
Ψ	stream function

ω	vorticity field
ω_1	vorticity field
$\omega_x, \omega_y,$	
ω_z	vorticity components in the x-y-z-directions
ω_w	vorticity at the wall
Ω	complex potential
V	volume of an uppermost bed particle and its surrounding fluid

1. INTRODUCTION

During the last 80 years much effort has been devoted to studies of local scour. Though great progress has been made many vital aspects of the local scour process still remain dim. The number of proposed design relations is quite substantial and the spread between various formulas is also considerable. Therefore no single scour formula has won general acceptance and the design of scour proof foundations still is made largely by judgment based on experience and precedent.

The main reason for the unsatisfactory knowledge about local scour is that this process involves two of the most intriguing problems in the field of hydraulics, three - dimensional turbulent boundary layers and sediment transportation mechanics.

Another difficulty is the lack of field data which depends on the tremendous practical difficulties in making observations during design conditions. Thus, most research has been carried out in laboratory scale. Therefore the crucial problem is to establish similarity relations in order to be able to interpret and extrapolate or generalize the laboratory data correctly but as long as the fundamental mechanisms are unknown it is virtually impossible to establish reliable similarity rules, especially as the range of variation of most of the parameters involved is very limited in the laboratory.

Though a full understanding of the phenomena involved in the scouring process seems remote, new experimental and analytical techniques have been developed which have increased the possibilities of revealing some of the inherent phenomena of the scouring process. Thus, there seems to be certain possibilities of replacing the purely empirical methods by semi-empirical.

Local scour, i.e. the local lowering of a streambed elevation, is caused by a disturbance of the sediment-transport capacity of the stream in a limited area such that the capacity to transport

is greater than the amount of sediment supplied. When an obstruction such as a pier is placed in an stream the flow pattern, and therefore the transport capacity, in the area adjacent to the obstruction is radically altered. As the bed material normally at rest is set in motion by the local increase in capacity, a scour hole develops, which in turn alters the flow pattern.

A full analytic solution would thus require a knowledge of (1.) the flow pattern around the obstruction, and the change of this pattern as the scour hole develops, and (2.) the relationship between the flow pattern along the bed of the stream and the resulting variation in transport capacity. However, in general no method exists for obtaining the necessary three-dimensional pattern for turbulent, boundary layer flow around an arbitrarily shaped obstruction, and furthermore such empirical sediment transportation relations as exist are not sufficiently rigorous to be applied to the problem at hand.

In this study the main objective has been restricted to a qualitative study of the flow patterns about cylinders and pipes over a plane boundary and the associated distributions of fluid forces acting on the bed. Thus, such results should primarily be applicable to scour protection design, where they can be combined with any preferred stability criterion to yield the necessary properties of the protective device. The results are also expected to provide a sounder starting point for limiting scour depth estimates.

2. PREVIOUS INVESTIGATIONS

2.1 Scour around bridge piers

One of the earliest investigations on scour and scour protection around bridge piers was a laboratory model study performed by Engels (1894). The pier shapes studied were circular, triangular and rectangular. Engels found that the maximum scour depth occurred at the upstream nose of the pier. This was contrary to what had earlier been anticipated, namely that the deepest scour occurred at the downstream end of the pier. The study on scour protection measures showed that a single and efficient protection could be provided by placing a horse-shoe shaped rock fill around the pier, with the open end downstream. However, no design relations concerning the required stone size or layer thickness for varying geometry or flow characteristics were worked out.

In 1911 Timonoff (1929) performed a test series at the hydraulic laboratory in Leningrad. He studied the importance of the spacing of bridge piers in the case of parallel bridges. He found that the upstream pier shelters the downstream pier if they are placed on the same axis and he therefore concluded that, new bridge piers in the case of parallel bridges should be in the immediate proximity of the old piers along the same axes. If the old bridge is considered safe, the new bridge should be built downstream of the old one but if the old bridge is considered unsafe, the new should be constructed upstream. Another result of the experiments was that it was found that the shape of the upstream end of the piers plays the most important part in scour. The shape of the downstream end has little effect on the scour action.

Schwartz (1929) studied the influence of sediment size on the depth of scour in the vicinity of bridge piers. In the experiments three sediment sizes, varying from 0.2 mm to 0.5 mm were tested. The pier was a pointed nose type of length 20 cm and width 4 cm. The depth was 5 cm and

the water velocity 40 cm/s which means that general bed load transport was established. Starting from an initially level bed, Schwartz found that the initial scour rate varied for the three sediment sizes but that the maximum observed scour depths were about the same. It was thus concluded that maximum scour depth was independent of sediment size. During these tests sediment was not recirculated.

Yarnell and Nagler (1931) studied the flow pattern and eddies around a standard North Carolina bridge pier and the resulting scour at the base of the pier. Starting from their conception of the obstruction caused by bridge piers, they made modifications of the basic pier which were also tested. They found the most efficient pier shape to have a lense shaped nose, straight sides and a semi-arc tail. A similar study was undertaken by Keutner (1932) who studied the flow around bridge piers of different shapes and its scouring effects. In concord with Engels' findings Keutner found maximum scour to occur at the upstream nose of the pier and he also found the downstream shape of the pier to have no influence on the depth of scour. The depth of scour was found to depend on the bluntness of the pier. The blunter the pier, the deeper the scour. A fish-shaped pier was found to cause 30 % less scour than a circular. Keutner also studied the effect of skewness that is when the approach flow is at an angle to the horizontal axis of the pier, and he found that at 27° skew the scour at both upstream and downstream ends were affected. Scour depth was twice that for 0° skew.

Applying the Bernoulli equation on the flow near a bridge pier, Fig. 2.1

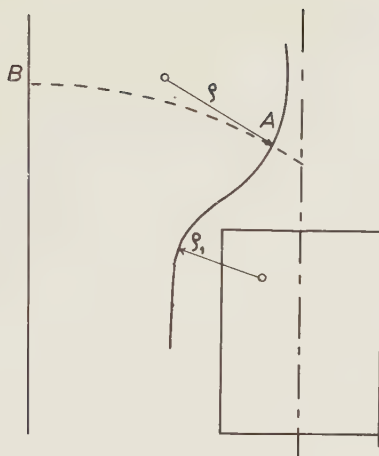


Fig. 2.1 Definition sketch for the Bernoulli equation

$$y_B + \frac{p_B}{\gamma} - \left(y_A + \frac{p_A}{\gamma} \right) = \frac{1}{g} \int_A^B \frac{V}{\rho} ds \quad (1)$$

where

y = elevation

p = pressure

γ = specific weight of fluid

V = local velocity

ρ = radius of curvature of streamline

ds = length element taken along a curve orthogonal to the streamline

Tison (1937) showed that when the flow is diverted by a bridge pier, the velocity must get a downward component. The magnitude of this component is shown to increase with the curvature of the streamline and with the non-uniformity of the vertical velocity distribution. The theoretical results were in qualitative accordance with experiments undertaken by Tison; a

rectangular pier (reduced values of ρ) produced a scour of 11.3 cm with a discharge of 30 l/s in a flume of width 70 cm and water depth 10.5 cm. The length of the pier was 24 cm and its width was 6 cm. A simple rounding of the piers reduced the scour to 9.1 cm whereas a triangular nose reduced it to 7.0 cm. An aerodynamic shape gave no further reduction, but the shape of a lens reduced the scour to 5.4 cm. The protection provided by a single pile before the lense shaped pier reduced the scour to 3.9 cm. This pile produced larger values of ρ and also gave a reduced width to length ratio.

A change of the velocity distribution affects the right hand member of the above Bernouilli equation and therefore also influences the scour depth. To investigate this effect the bed upstream of the pier was made rougher, which changed the velocity profile. The depth of scour around the lense-shaped pier was found to increase from 5.4 cm to 7.1 cm.

The theoretical analysis also implied that it should be possible to prevent the downward flow, and thereby to reduce the scour, by keeping the factor V^2/ρ constant. Therefore the velocity distribution was measured and the lense-shaped pier was given a variable radius of curvature corresponding to the values of V^2 . The result was a nonprismatic pier with a larger base. The scour depth around this model was less than 0.5 cm.

In connection with the Hardinge Bridge works Inglis, Thomas and Joglekar (1939) made model experiments on the piers. The models were made geometrically similar and the scales used were 1:40, 1:65, 1:105 and 1:210. Tests were made with the sand bed initially level and the smallest discharge was first tested. Then successively larger discharges were tested without releveing the bed. No sediment was recirculated. The different models gave similar scour depths for corresponding discharges and it was found that scour depth below the water level could be calculated from

$$\frac{D_s}{B} = 1.70 \left(\frac{q}{B} \right)^{2/3} 0.78 \quad (2)$$

where

D_s = scour depth below water level

B = width of the pier

q = discharge in cusec per ft upstream of the pier

In an additional study Inglis et al (1941, 1942) tried to develop criteria for depth of protective stone around the base of bridge piers. It was found that stones should be laid at the lowest practicable level to avoid the formation of high mounds of stone, which tend to obstruct the waterway and cause severescour downstream. As a rough estimate of the discharge causing failure at the nose of the pier the following formula was given

$$q_{lim} = 2.3 g^{1/2} (W_{ys})^{1/6} (s_s - 1)^{1/2} B^{1/12} H^{2/3} H_p^{1/4} \quad (3)$$

where

q_{lim} = limiting discharge per unit width

W = weight of protection per stone

γ = specific weight of water

s_s = relative density of stone

B = width of the pier

H = the depth of channel upstream of the pier

H_p = depth of stone below water level when failure begins

This formula implies that, other factors being the same, the higher the bed level upstream of the pier the more severe the attack. The greater the depth of the stone the more stable the protective layer.

Data from various Indian piers gave, Inglis (1949),

$$D_S = 2 \cdot 0.473(q/f)^{1/3} = 2D_L \quad (4)$$

where

D_S = depth of scour below water level

q = maximum discharge per ft width in cusec

f = silt factor = $1.76 \sqrt{m}$

m = mean size of sediment in mm

D_L = the Lacey regime depth i.e. the equilibrium depth for a straight channel reach

This formula was adopted for designing railway bridges in India. The bridge piers were designed with a grip length equal to half the depth of scoured bed below HHFL so that the total length of the piers below HHFL was $3 D_L$. No protection by means of stone pitching was provided. Observations of scour caused by the 1937 Connecticut storm were published by Stewart (1939). This report shows that many of the collapsed piers were originally created on sites considered to be safely out of the main channel. However, the cross-sections of the water courses could be drastically changed by the storm and some of the piers were undermined although there was no lowering of the mean bed level. These findings stress the importance of finding the appropriate reference level for design of foundations.

In 1948 a very ambitious research program was initiated at the Iowa Institute of Hydraulic Research. The results of this program, which included studies of the influence of pier shape, flow velocity, depth of flow, sediment characteristics and model-prototype comparisons, advanced the understanding of the local scour process to a remarkable extent. Laursen (1951, 1952, 1955, 1958, 1960, 1962), Laursen and Toch (1951, 1953, 1956), Laursen and Hubbard (1954).

Of great importance was the systematic application of the continuity concept to the bed sediment. In consequence with this, the local scour was considered not as one single process but as the net result of two opposed processes; sediment supply to the scour hole by general bed load movement, and sediment transport out of the scour hole caused by the flow in the scour hole.

This concept directly led to the separation of two fundamentally different scour processes:

- (1) Clear water scour, that is when there is no general bed load movement and thus no supply to the scour hole. In this case the scour is determined solely by the erosive capacity of the flow in the scour hole.
- (2) scour with continuous bed-load movement. In this case scour depth is determined by the relative transport capacities of the approach flow and of the flow in the scour hole.

In the clear water scour case the depth of scour increases until the maximum shear stress falls below some critical value. The strength of the flow in the scour hole was assumed to depend on the mean velocity and the pier shape. The limiting shear stress value was assumed to depend on the sediment characteristics.

In the continuous bed load case however, the influence of the abovementioned parameters was judged to be quite different. Laursen stated that a change of sediment properties is likely to have the same effect on the supply rate as on the rate of transport out of the scour hole. Therefore the limiting scour depth can be considered independent of sediment properties. Further, it was assumed that a change of mean velocity influenced the two processes in the same way, so that the balance was conserved. The depth of flow however, was assumed not to influence the strength of the flow in the scour hole, but only the transport

capacity of the approach flow. Thus the scour depth in the live bed case should depend only on the shape and width of the pier and on the depth of the approach flow.

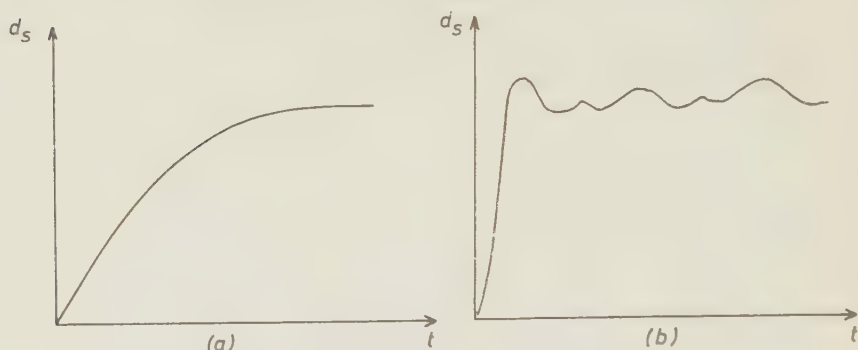


Fig. 2.2 Temporal evolution of scour
(a) clear water case
(b) continuous movement case

The two cases also have different time scales, which is indicated in Fig. 2.2. The transport capacity of the flow is fluctuating randomly and in the clear water case a continuously smaller percentage of the fluctuations contribute to the transport as the scour depth increases. This means that the scour depth is asymptotically approaching some limiting depth. In the continuous movement case however, the net outflow of sediment is determined by the balance or unbalance of two randomly fluctuating processes. This leads to a much faster development of the scour hole but the scour depth doesn't approach any limiting depth but fluctuates randomly around a mean value.

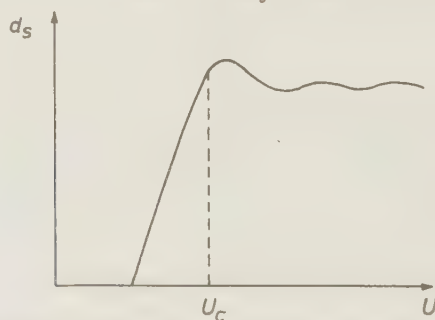


Fig. 2.3 Relation between depth of scour and velocity of the approach flow

The relation between depth of scour and velocity, according to Laursens' theory, is illustrated in Fig. 2.3. For sufficiently small approach velocities no scour takes place. As the velocity is increased, scour will develop and the equilibrium depth of scour increases with velocity until general bed load movement is established. After that further increase of the approach velocity does not affect the equilibrium scour depth. Because the establishment of bed load transport may be hindered by vegetation or by some other armour layer on the bed, the depth of scour may be somewhat greater than the equilibrium scour depth for the continuous bed load case.

Applying the above concepts and assuming that scour around bridge piers was similar to scour in a long contraction Laursen worked out the following design relations for rectangular piers aligned with the flow;

$$\frac{B}{H} = 5.5 \frac{d_s}{H} \left[\frac{\left(\frac{1}{12} \frac{d_s}{H} + 1 \right)^{7/6}}{\left(\tau_o / \tau_c \right)^{1/2}} - 1 \right] \quad (5)$$

where

H = depth of flow

d_s = depth of scour

τ_o = mean boundary shear of the oncoming flow

τ_c = the critical boundary shear

This formula is for the clean water case. The kinetic terms have been neglected and the ratio of the depth of scour at the pier and the depth of scour in the equivalent long contraction has been assumed equal to 12.

$$\frac{B}{H} = 5.5 \frac{d_s}{H} \left[\left(\frac{1}{11.5} \frac{d_s}{H} + 1 \right)^{1.70} - 1 \right] \quad (6)$$

This relation is for the case of general bed movement. In this

formula the depth of scour at the pier has been assumed to be 11.5 times the depth of scour in the equivalent long contraction.

Laursen and Toch have also worked out a design curve for rectangular piers and continuous sediment motion. The curve is given in Fig. 2.4

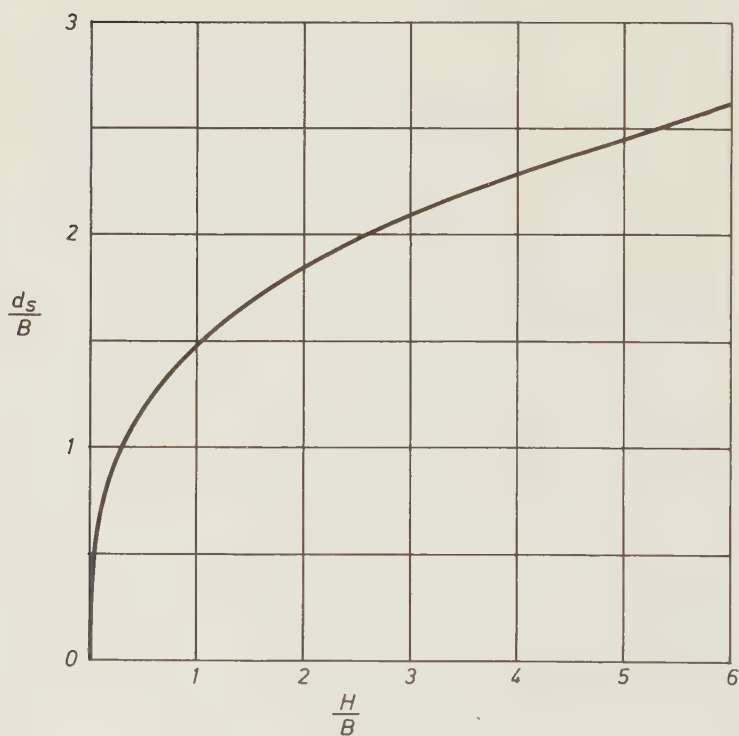


Fig. 2.4 Basic design curve for depth of scour. After Laursen and Toch (1956)

Though Laursen's assumptions about the influence of velocity and water depth are often questioned, the above design relations are those most commonly used besides engineering judgement [Klingeman (1973)].

Laursen and his collaborators also studied the influence of pier shape and angle of attack and worked out coefficients corresponding to these effects. Their results have, in principle, been confirmed by several investigators. Table 2.3 and Fig. 2.17. Their general conclusion is that pier forms, which could be expected to disturb the flow the most, cause the most severe scour. Laursen especially warns for the bad performance of piers, with a large length to width ratio, when they are attacked at an angle. The effects of streamlining are easily lost when the approach flow is skewed. Also the sometimes practised method of introducing a web between the upstream and downstream piers may lead to disastrous consequences. This point is also stressed by Romita (1960) in a discussion of Laursen's results. He points out that the collapse of many piers during floods is, in fact, to be ascribed more to the angles of attack which may occur due to unusual cross-currents and deformations of the bed, than to the unexpected intensity of the discharge. Laursen also studied the effect of scour arresting devices. He found that the vertical position of the arrestor was very important. If the arrestor was placed at bed level it must be very large to be effective. If it was made too small, scour would develop under the device. If the arrestor was placed below bed level the required size was diminished. Some of the devices tested are shown in Fig. 2.5.

To test the model-prototype relations, prototype data were collected from a carefully selected bridge having a pier which could be modelled simply, a deep sand bed at the pier, a constant angle of approach between the flow and pier at all stages, and a crossing geometry such that local rather than general scour would be expected. This bridge had a single center pier well aligned with the flow, which approached the pier from a long straight

reach. Models of the prototype pier were built at two scales and tested in the laboratory at a constant velocity on a single sand bed, with depth of flow varied from run to run.

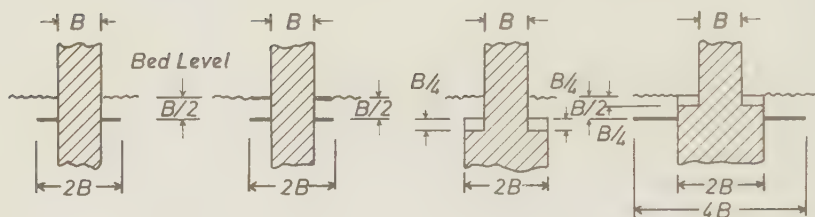


Fig. 2.5 Scour arresting devices tested by Laursen and his collaborators

Model-prototype conformity was checked for three floods. Reasonable agreement of results was obtained and the comparison was interpreted to show that geometry was the controlling influence over the depth of scour. This meant that scour depth comparisons between model and prototype could be treated like any other length parameter.

Another important contribution to scour studies was provided by Chabert and Engeldinger (1956) who collected an extensive set of data. Although their attempts to find a similarity relation failed, they showed that as long as the approach velocity is less than the velocity starting bed load movement, U_c , the depth of scour increases with the velocity. For velocities higher than U_c the depth of scour was found to be independent of velocity but somewhat smaller than the depth at U_c . These results agree with Laursen's findings.

Contrary to Laursen, Chabert and Engeldinger concluded that for water depth greater than the width of the pier, water depth does not affect the depth of scour. Also Laursen's theory about the negligible importance of sediment properties is contradicted by some results of Chabert and Engeldinger. Some results are given below.

Table 2.1 Maximum depths of scour observed by Chabert and Engeldinger around circular piers (in mm)

	Pier diameter mm	50			100			150		
		100	200	350	100	200	350	100	200	350
Sediment size mm	3	95	87	80	149	155	165	197	210	215
	1.5	116	124	114	156	191	197	210	226	233
	0.52	108	119	102	133	165	147			
	0.26	86	77	71	108	109	113			

From Table 2.1 it can be seen that there is an obvious shift in scour depth between the different sediments. The smaller scour depths for the 0.26 mm sand may be due to the smaller voids ratio, which was 0.3 for this sand and 0.4 for the others. Some of the remaining variation is certainly random but it still seems that some sediment properties influence the limiting scour depth in a yet unknown way.

The influence of the water depth is difficult to determine. Chabert and Engeldinger concluded that for water depths less than the diameter of the pier water depth may limit the depth of scour but for water depths greater than the diameter, the maximum scour becomes independent of water depth. Another possible interpretation of table 2.1 is that the depth of maximum scour increases with water depth until the water depth is approximately equal to four times the pier width, thereafter the depth of scour decreases with further increase of water depth.

These points are further illustrated in Figs. 2.6 and 2.7

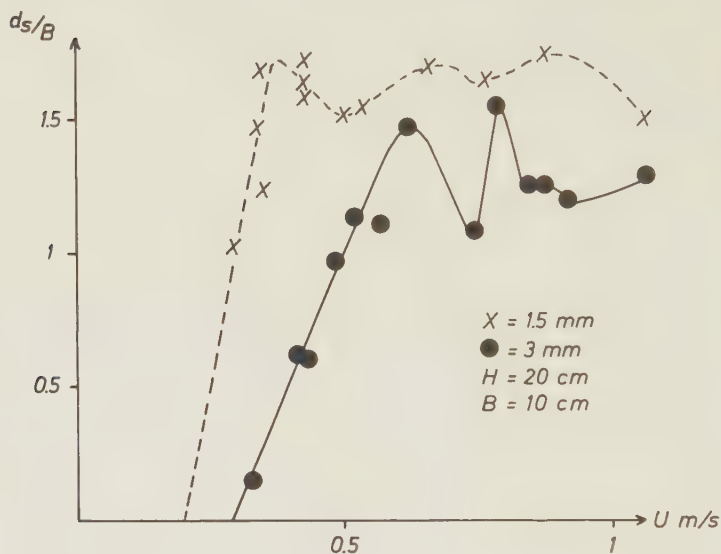


Fig. 2.6 Illustration of shift of scour depth with sediment properties. (From Chabert and Engeldinger's results)

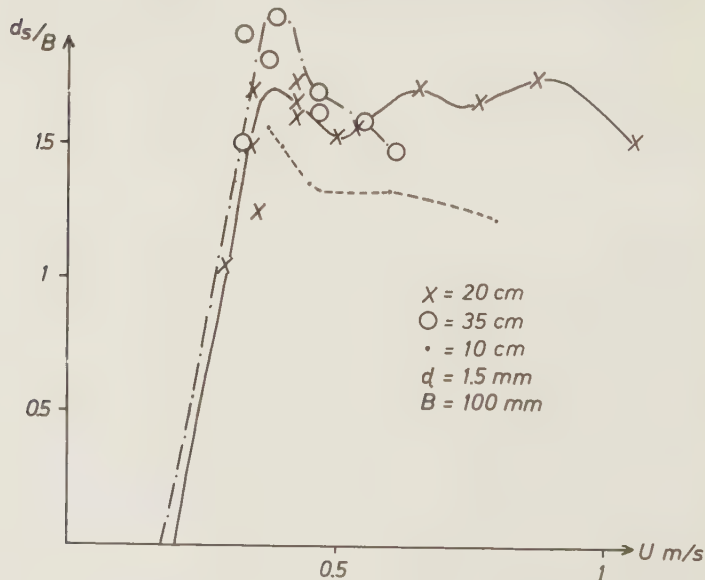


Fig. 2.7 Illustration of shift of scour depth with depth of flow. (From Chabert and Engeldinger's results)

From these diagrams it can also be seen that Laursen's design curve, which predicts a unique relationships between d_s/B and H/B does not hold for these experiments.

The effects of pier shape and of angle of attack were also investigated. The findings were in qualitative agreement with the results of other investigators. See Table 2.3 and Fig. 2.17.

They also studied some scour protecting devices. The most effective arrangement consisted of three small diameter piles arranged in a triangular pattern upstream of the pier to be protected. The efficacy of such a device depends on the spacing but for optimal spacing the scour depth was reduced by 50 %.

Other arrangements tested were a cylindrical pier placed on a concentric circular foundation and a cylindrical pier provided with a thin circular collar. During these tests the diameters of the foundation or the collar were varied as well as the vertical position, relative to bed level, of the top of the foundation and the collar respectively.

It was found that, in order to be effective the devices must be placed below bed level unless their diameters are very large compared to the diameter of the pier to be protected. The optimum distance was found to be about one pier radius below bed level, Fig. 2.8 and 2.9. These results agree very well with those of Laursen's.

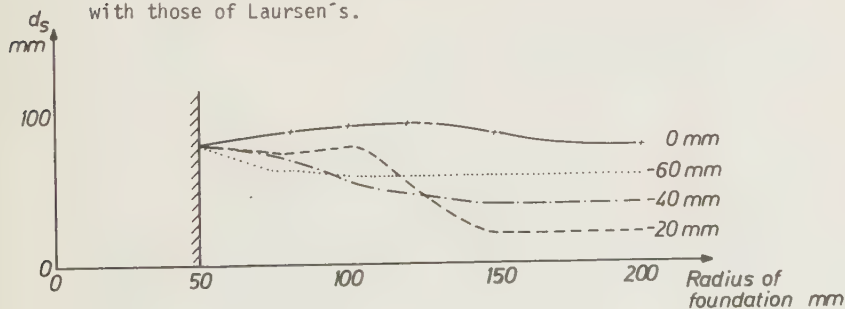


Fig. 2.8 Variation of maximum scour depth with diameter and vertical position of foundation. [From Chabert and Engeldinger(1956)]

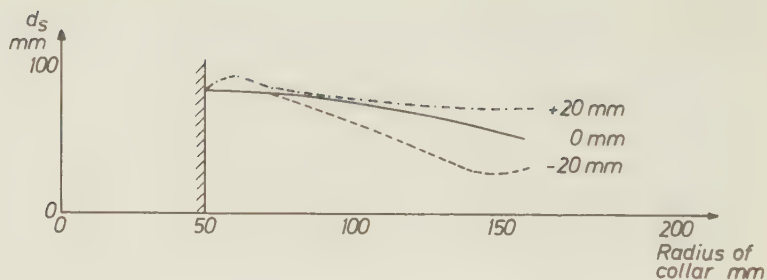


Fig. 2.9 Variation of maximum scour depth with diameter and vertical position of protective collar.
[From Chabert and Engeldinger (1956)]

A small perturbation method for computation of the energy of the secondary flow around struts and airfoils was presented by Hawthorne (1954). The method works only for streamlined shapes but nevertheless Hawthorne was able to show that scour depth increases with the energy of the secondary flow. However, he also found that the energy of the secondary flow is not strong enough to account for the extra scour and he found it more likely that the modification of the primary flow by the secondary increases the scouring capacity.

Following a somewhat different line of approach Lighthill (1956a, 1957) was able to make an exact computation of the secondary flow, created in an ideal fluid flowing past an infinite cylinder of arbitrary cross-section, if the cylinder is at right angles to the oncoming flow, and if this flow has a small and constant velocity gradient along the cylinder.

Allowing for a slight viscosity but restricting the analysis to the plane of symmetry, Toomre (1959) showed that, for a circular cylinder or for a flat plate at right angles to the flow, Lighthill's solution is valid except in a region very close to

the obstruction, where Lighthills solution does not satisfy the no-slip condition but gives infinite velocities at the wall. Inspired by the results of Lighthill and Toomre, Moore and Masch (1963) presented a qualitative description of the scouring mechanism. Their idea was that the difference in stagnation pressure on the upstream side of the pier creates a vertical flow, which creates a horseshoe vortex as it hits the channel bed. Thus if this downward flow could be prevented, the scour would be reduced. They presented some devices meant to prevent this downward flow, but no experimental results were given.

The same idea is also presented by Thomas (1967) who tried to reduce scour by putting a circular collar around the pier shaft. The effect by the collar was found to be small unless the collar was placed below the original bed level. Similar ideas were also presented by Tanaka and Yano (1967) who also tried to reduce the secondary flow by releasing the pressure by a hole through the pier at bed level. This hole was shown to have only a minor influence on the depth of scour. Further Tanaka and Yano investigated the importance of the height of the pier. They found that, as long as the pier penetrated the layer of large velocity gradient, the height of the pier did not considerably influence the depth of scour. In a reversal of this experiment they tested a 'floating' pier. This test showed that unless the bottom of the pier was very close to the bed, the scour was negligible. Even when the bottom of the pier was at bed level, the scour was considerably less than for a pier protruding into the bottom. They concluded that the vertically downward flow along the surface of the pier seems to be generated secondarily by the vortex motion and does not seem to affect directly the local scour around the pier of small diameter.

Shen, Karaki, Schneider and Roper (1966) tried to treat the scour mechanism analytically. Their study was concerned only with blunt nosed pier shapes, that is pier shapes creating a horseshoe vortex at the base of the pier. The momentum approach was

used to estimate the strength of the horse-shoe vortex. Basing their analysis on the following assumptions;

- (1) The flow and vortex system can be treated as steady.
- (2) There is no shear at the upper edge of the boundary layer.
- (3) The oncoming flow is two-dimensional and obeys the potential flow theory.
- (4) The boundary-layer thicknesses in the radial and tangential directions are equal.
- (5) The velocity distribution in the radial direction is given by

$$u_r = u_0 \left(1 - \frac{z}{\delta}\right) \left(\frac{z}{\delta}\right)^{1/7} \quad (7)$$

and in the tangential

$$u_\theta = U \left(1 + a^2/r^2\right) \left(\frac{z}{\delta}\right)^{1/7} \quad (8)$$

where

- u_0 = a characteristic velocity
- r, θ, z = cylinder coordinates
- a = half-width of the pier
- δ = height of boundary layer
- U = the approach velocity

- (6) The shear stress on the bed is given by

$$\frac{\tau_0}{\rho} = 0.0225 \left(\frac{\nu}{\delta}\right)^{1/4} (u)^{7/4} \quad (9)$$

where

- τ_0 = shear stress of the approach flow
- ρ = fluid density
- ν = kinematic viscosity

$$u = (u_r^2 + u_\theta^2)^{1/2}$$

- (7) The no-slip condition applies at the bed.
- (8) The pressure distribution in the neighbourhood of maximum scour is given by the potential flow theory.
- (9) The characteristic velocity, defined in Eq. (7), is proportional to the undisturbed mean velocity of the oncoming flow, i.e.

$$u_0 \propto U \quad (10)$$

- (10) The flow is restricted by the shear force at the bed.
- (11) The momentum equations for the boundary layer can be solved at $\theta = \pm 45^\circ$. It is assumed that the resultant shear stress at $\theta = \pm 45^\circ$ is nearly equal to the shear stress at $\theta = 0^\circ$ because the scour depth is almost the same.
- (12) The approach applies only to the region near the cylinder where the momentum of the incoming flow is assumed to have no effect.

They found that the strength of the vortex should be proportional to the product of pier width and approach velocity. According to their conception of the problem, the maximum scour depth should depend on the strength of the vortex, and as they judged the viscosity ν , to be an important parameter, they concluded that the maximum scour depth should be a function of the pier Reynolds number, $Re = 2aU/\nu$. By means of regression analysis they found that

$$(d_s)_{\max} = 0.000292 Re^{0.645} \quad (11)$$

where

$$(d_s)_{\max} = \text{maximum scour depth in ft.}$$

should be the envelope of the scour data considered. In a later paper, Shen et al (1969), the formula was given as

$$(d_s)_{\max} = 0.00073 \text{ Re}^{0.619} \quad (12)$$

The study also included extensive tests on scour protection methods. Principally, two ideas were tested;

- (1) to prevent or hinder the development of the horse-shoe vortex.
- (2) to confine the vortex to a small, protected area.

The accomplishment of the first idea was attempted by placing a smaller pier upstream of the main pier. The method worked well if the piers were correctly spaced and well aligned with the flow. This is in accordance with the findings of Timonoff (1929) and Tison (1939). Roughness elements on the upstream side of the pier, to weaken the vertical downflow, were shown to have no effect and like Tanaka and Yano (1967), Shen et al found that the strength of the secondary flow could not be weakened by a slot in the pier.

The confinement of the vortex was attempted by means of a horizontal plate at the base of the pier. The plate was given a vertical lip at the outer edge to prevent the vortex from spreading beyond the plate. Shen et al found, like many previous investigators, that the vertical position of the plate is extremely important. If the device is placed at too high a level, scour will continue beneath the plate as if no plate existed. In some cases, the scour could be even worse than without the plate.

Using the data of Chabert and Engeldinger (1956) and some field measurements Larras (1963) tried to work out a formula for the deepest scour, that could be created by a given pier. He found

this depth to be given by

$$d_s = \frac{10}{3} K_1 K_2 B^{3/4} \quad (13)$$

where

d_s = maximum maximum of scour depth in cm

K_1 = a factor depending on pier shape

K_2 = a factor depending on angle of attack

B = pier width in cm

The values of K_1 and K_2 proposed for different shapes and angles of attack are given in Fig. 2.17 and Table 2.3.

Breusers (1972) supported the idea that for the case of continuous bed load transport, and for water depths greater than twice the pier width, the maximum scour depth is a function of pier shape and width only. For circular piers he proposed

$$d_s = 1.3 - 1.5B \quad (14)$$

This formula was based on laboratory as well as field measurements.

Breusers made a reservation for the influence of sediment characteristics; if to large a portion of the sediment load is carried in suspension, the above formula may well underestimate the depth of scour.

Hancu (1971) studied the influence of water depth. Like Chabert and Engeldinger he found that as long as the water depth is greater than the pier width, the depth of scour is independent of water depth. Smaller water depths tend to reduce the depth of scour. He also studied the clearwater scour case, and found that for circular piers scour begins when the approach velocity reaches $U_c/2$, regardless of pier size. The greatest scour depth

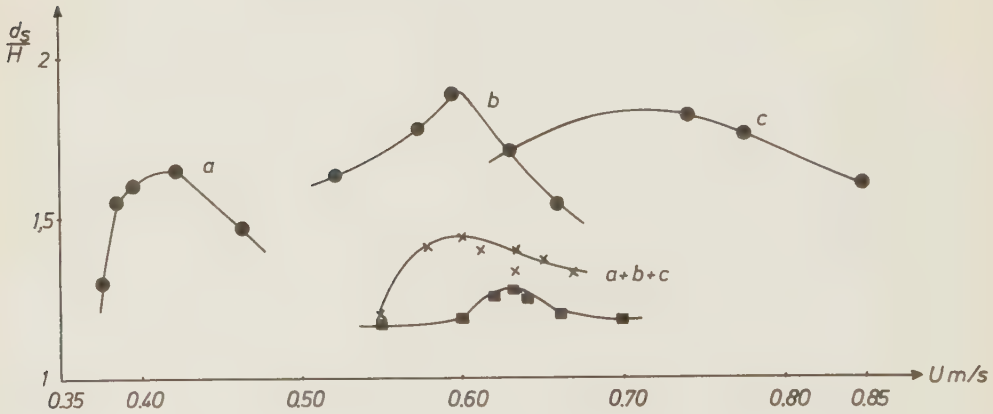
was observed for $U = U_c$. For approach velocities between $U_c/2$ and U_c the scour depth varies linearly with velocity

$$d_s = \left(\frac{2U}{U_c} - 1\right) d_{s_{\max}} \quad (15)$$

the maximum scour depth was found to be given by

$$d_{s_{\max}} = 2.42 \left(\frac{U_c^2}{gB}\right)^{1/3} \quad (16)$$

In a study of the effects of sediment size and gradation, Nicollet and Ramette (1971) found that for near uniform sand, the maximum scour depth was independent of sediment size. When the experiments were repeated using a mixture of the sediments of the previous tests, the maximum scour depths were considerably reduced



- a 0.77 mm < d < 1.12 mm no sediment supply
- b 1.62 mm < d < 2.24 mm " " "
- c 2.77 mm < d < 3.17 mm " " "
- a + b + c (upper) 1/3 a + 1/3 b + 1/3 c no sediment supply
- a + b + c (lower) 1/3 a + 1/3 b + 1/3 c continuous sediment supply

Fig. 2.10 Influence of sediment gradation according to Nicollet and Ramette (1971)

2.2 Scour around pipes

Pipeline scour has been less attended to than bridge scour. Previous research has primarily attempted to determine the hydraulic forces on the pipe itself. One among few early studies is reported by Rawn and Bowerman (1950). They identified two causes of scour;

- (1) water, sprinkling through a leaking joint, washes the surrounding sand away and a small hole is created under the pipe. Underflow caused by cross currents may then enlarge this hole laterally until the pipeline loses its support and collapses.
- (2) currents along the pipe are disturbed by the joints. The vortices, thus created, may erode the bed in the neighbourhood of the joints. The initial scour holes may then be enlarged by cross-currents. As in the previous case, this may lead to the collapse of the pipe.



Fig. 2.11

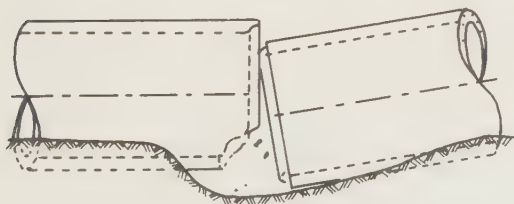


Fig. 2.12

Fig. 2.11 Small scour hole caused by leaking joint

Fig. 2.12 Scour hole enlarged by cross-currents

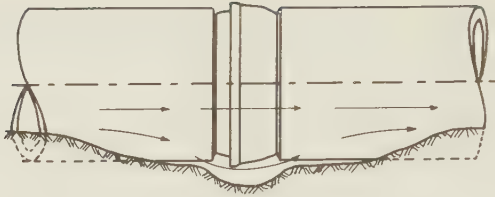


Fig. 2.13

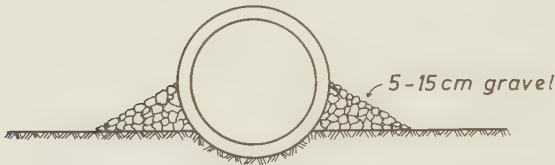


Fig. 2.14

Fig. 2.13 Vortices created at a joint by current parallel to pipe

Fig. 2.14 Scour protection suggested by Rawn and Bowerman (1951)

5 - 15 cm riprap was found to provide a satisfactory protection against both types of scour. It was recommended that the riprap should be extended to the spring of the pipe.

Their findings were partly confirmed by Carstens (1966). He found that scour always started from a point of local failure of the underground. From there the scour was extended laterally until the whole line was undercut. Carstens tried to relate the extent of scour to a sediment number

$$N_s = U / [(r_s - 1)gd]^{1/2} \quad (17)$$

where

d = the size of sediment

He proposed the functional relationship

$$d_s/L = f \left[(N_s^2 - N_{sc}^2)^{5/2} \left(\frac{d}{L} \right) \left(\frac{Ut}{L} \right) \right]$$

where

L = a characteristic length h

t = time

N_{sc} = the sediment number for incipient motion

Lately, there has been a noticeable increase of interest in pipeline scour. Among later studies are; Hydraulics Research Station Wallingford (1973); A field study on the behaviour of pipes laid on the sea floor. It was found that considerable undercutting occurred over greater parts of the pipe lengths. The pipes rested in trenches and were supported on sand bars at regular intervals. The bed profiles upstream of the pipes were generally fairly steep, approximately equal to the angle of repose of the sand. Downstream the slopes were more gentle and sand had been deposited at some distance from the pipe.

A flexible pipe was also tested. With this pipe hardly any spanning occurred. The pipe settled gently and uniformly along its length until a stable position was reached. The pipe elements tested were of limited length and the deepest scour was found near the ends of the pipes. This is perfectly in concord with the results of Carstens.

Kjeldsen, Gjörsvik and Bringaker (1974) studied scour patterns around pipes, resting on the channel bed or partly embedded in it. Their findings about scour patterns and scour development are summarized in Fig. 2.15

Thus it was found that if the pipe originally is at least half-way embedded in the sand, the risk of undercutting is small.

For estimation of the scour depth, the following formula was worked out;

$$d_p = \left(\frac{u^2}{2g} \right)^{1/5} \cdot B^{4/5} \quad (18)$$

where

d_p = equilibrium scour depth below the bottom of the pipeline

B = pipe diameter

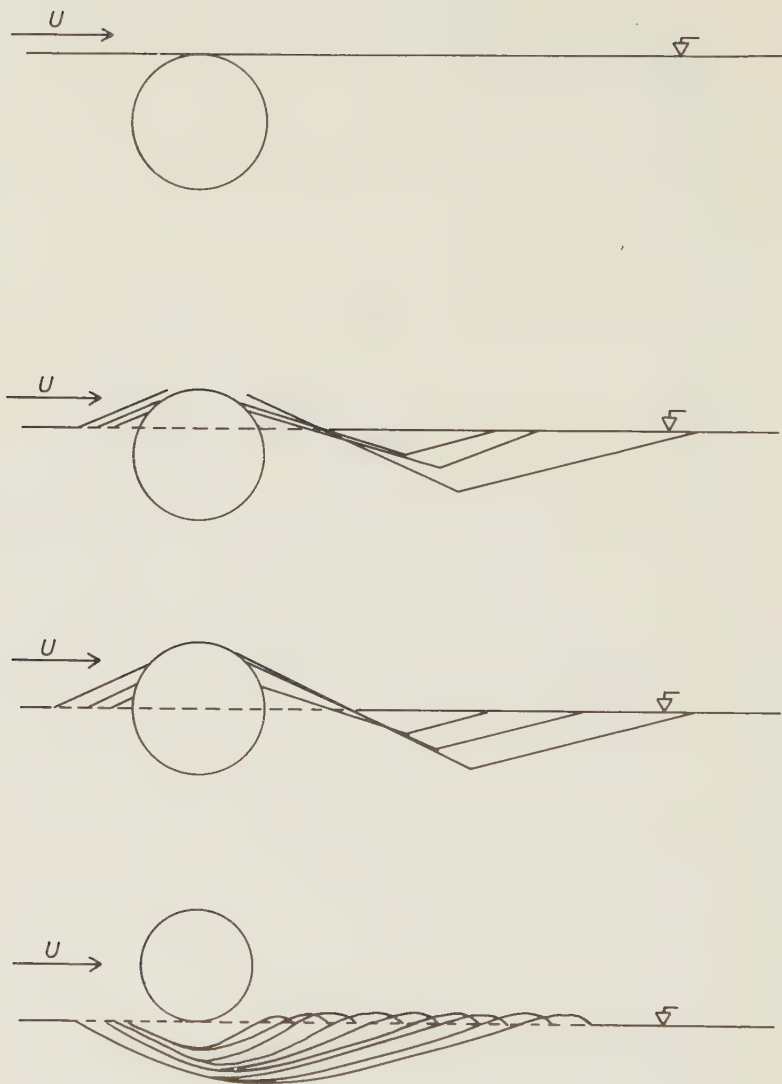


Fig. 2.15 Generalized scour profiles near pipelines.
After Kjeldsen et al (1974)

Thus, the equilibrium depth of scour was found to depend mainly on the geometry of the structure and to a small extent on the flow velocity.

The ratio of water depth to pipe diameter was judged to have no influence if this ratio is greater than 3 - 5. As a consequence the structure Froude number was found to give a much better correlation with scour than the overall Froude number for the flow.

This indicates very clearly that it is the local flow picture around the structure and the local turbulent distribution that is responsible for the scour.

Kjeldsen et al therefore concluded that no further development can be expected using overall parameters for the flow picture, and the need for additional analysis using data on local turbulence intensities around the structure was underlined.

During the tests no influence of relative grain sizes was found.

The sediment used in these tests was quite fine but from other investigations Kjeldsen et al found that no influence of grain size can be expected for materials with $d_{50} < 0.5$ mm, where d_{50} represents the grain size for which 50 % by weight, of the sediment is finer.

Townsend and Farley (1973) found that the rate of scour, around a submarine pipeline crossing, increased locally after the bed had been scoured to the top of the pipe. Inspired by a result of Vance who found that drag and lift forces increase if the supporting mobile bed is replaced by a solid bed or if the surface of the pipe is made smoother, they tried to hinder undermining by placing a vertical fin on the top of the pipe. Tests showed that such a modification in fact enhanced the chances of

the foundation to withstand a washout.

They also investigated the importance of bearing pressures. By comparing runs with similar Froude numbers, they concluded that installations with lower bearing pressures were characterized by a much faster and sudden washout.

The tests also showed that scour would occur on both sides of the pipe until the nondimensional shear stress

$$\tau_o^* = \tau_o / (s_s - 1)gd \quad (19)$$

was close to 0.06. When the bed was scoured below the spring of the pipe, the bed would be suddenly washed out.

If a protection of crushed rock was placed on both sides of the pipe, the general bed scour could extend to an elevation below the bed of the pipe installation without a washout occurring.

The crushed rock dykes appeared to be stable if the Froude number (with the rock size as length parameter) of the flow over the pipe did not exceed 0.5.

For translation of model results to prototype conditions two methods were suggested;

(1) Froude number similarity: $\frac{U_m^2}{gd_m} = \frac{U_p^2}{gd_p}$

where the riprap size is used as characteristic length.

Index m refers to model conditions and index p to prototype

(2) equality of dimensionless shear stress: $\tau_{om}^* = \tau_{op}^*$

Among the most renowned studies on the initiation of motion are those of Shields (1936) and White (1940). Both of them considered the balance of forces acting on a grain resting on a cohesionless bed.

Shields used dimensional analysis to define relevant parameters and by extrapolating observed relations between these parameters he arrived at a formula for the critical tractive force, τ_c .

$$\tau_c = f\left(\frac{u_* d}{\nu}\right)(\rho_s - \rho)gd \quad (22)$$

where

$$u_* = (\tau_o/\rho)^{1/2}$$

ρ = fluid density

ρ_s = solid density

d = bed sediment size

White tried to evaluate the forces acting, and arrived at

$$\tau_c = \frac{B^2}{w} (\rho_s - \rho)gd \quad (23)$$

where

$$\frac{B^2}{w} = \frac{r\pi \tan \phi}{6}$$

r = a packing factor

ϕ = angle of repose of the sediment

with
$$\frac{B^2}{w} = f\left(\frac{u_* d}{\nu}\right)$$

equations (22) and (23) become identical.

Empirical evaluations of the functions B_w or $f(\frac{u_* d}{v})$ show considerable scatter. White explained the scatter as a matter of turbulent properties of the flow. However there are several sources of scatter. The formulas are oversimplified and take no account of uplift due to the nearness of the boundary and the strong velocity gradient, which tends to move any submerged body towards the faster moving fluid layers. Nor do the formulas account for graded sediments or channel configuration.

Sediment transport formulas have been derived by among others, Shields (1936), Einstein (1942, 1950), Einstein-Brown (1950), Kalinske (1947), Laursen (1958), Meyer-Peter (1948) and Englund-Hansen (1967). As stated above, most of the formulae are related to the Du Boys formula. Because of the practical difficulties, very few systematic field measurements have been made of sediment transport rates. However, Vanoni and others (1961) have compared field results with an exhaustive series of results calculated from many of the established sediment transport formulae. The amount of scatter in field measurements is quite important, and the difference between the various formulae is equally extreme. Fig. 2.16.

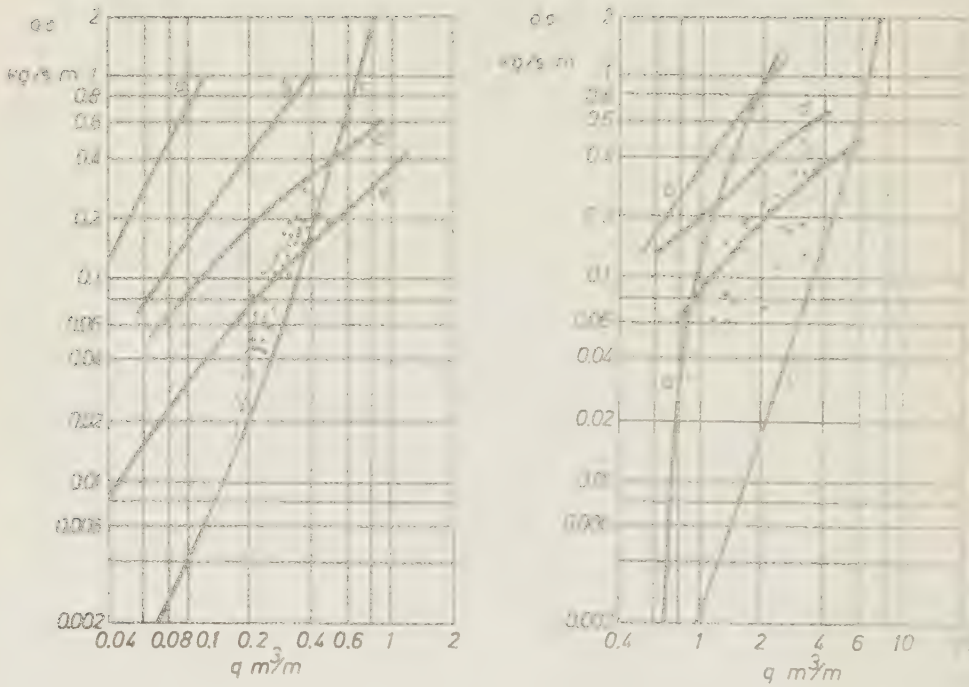


Fig. 2.16 Sediment rating curves and observed values in natural streams, after Vanoni et al (1961). Left; Niobara River, $S = 0.00129$, $m = 0.283$ mm Right; Colorado River at Taylor's Ferry, $S = 0.000217$, $m = 0.320$ mm
a. Laursen; b. Shields; c. Einstein's bed load function; d. Du Boys; e. Einstein-Brown

Egiazaroff (1967) stresses that a uniform sediment can never reproduce the behaviour of a graded material. More parameters than the mean diameter or the fall velocity are needed to characterize a given bed sediment. Egiazaroff also states that one of the primary causes for discrepancies between measured and calculated sediment transport rates is the definition of shear stress τ_0 or shear velocity u_* used in the formulas. In reality, changes of the initially flat bed into a rippled bed leads to a rapid increase of the friction factor, while the mean velocity changes relatively little. However, only a portion of this increased friction, or shear, is participating in the sediment transport process. The other part of the resistance is form resistance, caused by the bed forms. Thus there is an obvious need to separate these two resistance types, in order to be able to predict sediment transport correctly. Such a separation was attempted by Einstein and Barbarossa (1952). They supposed that the total shear could be separated into two components, τ_0' and τ_0'' , each corresponding to a component of the flow area, A' and A'' , so that

$$\tau_0' = \gamma A' S / P = R' S, \quad \tau_0'' = \gamma A'' S / P = R'' S \quad (24)$$

where

P = the hydraulic radius

P = the wetted perimeter

It was then assumed that the sediment transport, and bed formation depended on τ_0' alone. Adopting Shields' bed load function meant that the resistance coefficient due to bed forms should be a function of $\tau_0' / \gamma d (s_s - 1)$. This coefficient was also found to be a function of U / u_*'' .

Thus

$$\frac{U}{u_*''} = \frac{U}{(g R'' S)^{1/2}} = f \left[\frac{d (s_s - 1)}{R' S} \right] \quad (25)$$

This equation has found some experimental support.

Engelund (1966) tried to accomplish the separation in a somewhat different way. He assumed that a certain regime of a sediment carrying river can be characterised by three parameters.

- (1) the slope
- (2) the ratio of water depth to sediment size H/d
- (3) the dimensionless effective boundary shear

$$\theta' = \tau_0' / (s_s - 1) \gamma d_{35} \quad (26)$$

where

d_{35} = the grain size for which 35 % by weight is smaller

Engelund admitted that the relative density, s_s , of the grains also has some influence. However in practice it is hardly variable.

Plots of H/d_{35} against θ' for some American rivers showed that for fixed slope

$$H/d_{35} = C(\theta')^{1/2} \quad (27)$$

The value of the dimensionless constant C is determined by the value of H/d_{35} for $\theta' = 1$.

The slope of the energy line may be approximated by the mean bed slope over the channel reach in question. Such an approximation has also been suggested by O'Brien and Rindlaub (1934).

All the above mentioned studies deal with the temporal mean of the boundary shear. However, grains are dislodged not by the exertion of a steady fluid force but by the action of isolated fluid disturbances, which we in line with normal practice can describe as eddies. The grains themselves are not uniformly susceptible to movement but have a randomly distributed resistance to dislodgement.

Now if there are a number of eddies of varying strength and a number of grains of varying stability, movement will occur when a strong enough eddy meets an unstable enough grain. As the strength of the flow increases, the number of such encounters will rise and thus increase the transport rate. This mechanism is essentially statistical in nature, and therefore it is difficult to predict the relationship between bed load and the strength of flow.

This mechanism was first treated by Einstein (1942) who expressed the probability of movement of any one particle in terms of the weight rate of sediment transport, the size and immersed weight of the particle, and a characteristic time which is taken to be a function of the quotient of the particle size divided by its fall velocity. It was postulated that a given particle size moves in a series of steps, and that a given particle does not stay in motion continuously, but is deposited on the bed after a few steps. The number and volume of such moving particles yields, then, the rate of sediment transport.

The probability of movement can also be estimated hydraulically from a mechanical model, in which the lift due to differences in pressures correlated with the turbulent pulsations, is the active agent, with the effective velocity at the nominal limit of the laminar layer as main parameter.

If the values of this probability, as obtained from the two alternative methods are respectively ϕ and Ψ , Einstein obtained the result

$$0.465 \phi = e^{-0.391 \Psi} \quad (28)$$

The function relating ϕ and Ψ has to be obtained experimentally. The analytical model does not explain the mechanism of the transport problem although various concepts of fluid mechanics are used to justify the steps of its derivation. Fundamentally

the result is not superior to empirical formulae.

Grass (1970) used visualization techniques to look into the statistic properties of the pick-up mechanism. From the observed velocity profiles, he was able to calculate the instantaneous boundary shear stress τ_0 . He also observed the instantaneous shear stress τ_c , acting on a grain at the moment it was dislodged. Thus he was able to establish two shear stress distributions, one related to the strength of the flow and the other to the stability of the grains.

Over the two fold range of sediment sizes, used by Grass, the average instantaneous critical shear stress, $\bar{\tau}_c$, shows little variation, Table 2.2.

Table 2.2

Mean diameter of sand mm	0.090	0.090	0.115	0.138	0.165	0.195	0.143
Average bed shear stress $\bar{\tau}_0$ N/m ²	0.206	0.191	0.204	0.192	0.185	0.226	0.134
Standard deviation $\sigma_{\tau_0} / \bar{\tau}_0$	0.422	0.402	0.354	0.397	0.346	0.349	0.469
Average critical bed shear stress $\bar{\tau}_c$	0.254	0.224	0.244	0.212	0.249	0.250	0.210
Standard deviation $\sigma_{\tau_c} / \bar{\tau}_c$	0.353	0.334	0.259	0.287	0.246	0.324	0.325

Table 2.2 Critical shear stress versus grain diameter after Grass (1970)

According to Grass, a possible reason for this is the variation of the height of the tops of the grains above the level of zero velocity. This height was found to increase in direct proportion

to the grain diameter. For a particular value of instantaneous bed shear stress, therefore, the larger grains were subject to a greater shear stress intensity than were the smaller grains.

The measured distributions of $\tau_0/\bar{\tau}_0$ were, according to Grass, in close agreement with results obtained by Compte-Bellot, Schraub and Kline, and Clarke. Their studies provide strong confirmation that over a wide range of experimental conditions, the ratio $\sigma_{\tau_0}/\bar{\tau}_0$ is constant at 0.4 for twodimensional channel flows. (σ_{τ_0} is the standard deviation in the distribution of τ_0 values around the mean $\bar{\tau}_0$).

The critical flow condition can be defined in terms of the critical shear stress distribution of the bed material;

$$\bar{\tau}_0 + n_1 \sigma_{\tau_0} = \bar{\tau}_c - n_1 \sigma_{\tau_c} \quad (29)$$

where

n_1 = simply a multiplication factor

The experimental results indicated that

$$\sigma_{\tau_c} \approx 0.3\bar{\tau}_c, \quad \sigma_{\tau_0} \approx 0.4\bar{\tau}_0 \quad (30)$$

Substitution of these values into the above formula yields

$$\bar{\tau}_0 = \frac{\bar{\tau}_c(1 - 0.3n_1)}{1 + 0.4n_1} \quad (31)$$

According to the experiments $n_1 = 0.625$ yields a satisfactory criterion for the initial movement of sands with diameter less than 0.25 mm over a range of water temperatures between 0° C and 30° C. A small amount of bed movement was observed for bed shear stresses corresponding to the $n_1 = 0.625$ criterion. Almost complete bed stability was found to correspond to a value of $n_1 = 1$.

2.4 Effects of pressure gradients and seepage forces.

These effects have not, with a few exceptions been much attended to. Posey (1939, 1961, 1963, 1974) however, has repeatedly urged the importance of these effects being taken into account. According to him, pressure gradients, induced by surface waves or other flow disturbances, may generate a flow through the bed. The associated seepage forces make the bed much more susceptible to erosion. To support his theory Posey refers to cases where pervious scour protections have been found to behave much more satisfactorily than impervious.

This idea is supported by O'Brien and Morison (1952) who, in connection with a study of wave forces, stated that pressure fields associated with breakers must extend some distance into a porous bed and cause flow into and out of the bed. The upward flow probably fluidizes a large amount in addition to that which is visibly lifted. This situation may result in movement of the upper portion of the bed en masse, a mechanism fundamentally different from that of bed load transport in steady flow.

By simply substituting the Darcy seepage equations into the two-dimensional equation of continuity, Sleath (1970) arrived at

$$\frac{\partial^2 p}{\partial x^2} + \frac{k_y}{k_x} \frac{\partial^2 p}{\partial y^2} = 0 \quad (32)$$

where

k_x, k_y = the permeability coefficients for the
x- and y- directions

$$y = 0, p = p_0 \cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{L}\right) \quad \text{at the bed}$$

$$\text{and } v = 0 \text{ at } y = -D$$

where

T = wave period

L = wave length

he found the solution

$$p = p_0 \frac{\cosh \frac{2\pi}{L} \left(\frac{k_x}{k_y} \right)^{1/2} (y + D)}{\cosh \frac{2\pi}{L} \left(\frac{k_x}{k_y} \right)^{1/2}} \cos 2\pi \left(\frac{t}{T} - \frac{x}{L} \right) \quad (33)$$

Thus

$$u = - \frac{p_0 k_x \frac{2\pi}{L}}{\mu} \frac{\cosh \frac{2\pi}{L} \left(\frac{k_x}{k_y} \right)^{1/2} (y + D)}{\cosh \frac{2\pi}{L} \left(\frac{k_x}{k_y} \right)^{1/2}} \sin 2\pi \left(\frac{t}{T} - \frac{x}{L} \right) \quad (34)$$

$$v = \frac{p_0 (k_y k_x)^{1/2} \frac{2\pi}{L}}{\mu} \frac{\sinh \frac{2\pi}{L} \left(\frac{k_x}{k_y} \right)^{1/2} (y + D)}{\cosh \frac{2\pi}{L} \left(\frac{k_x}{k_y} \right)^{1/2}} \cos 2\pi \left(\frac{t}{T} - \frac{x}{L} \right) \quad (35)$$

Sleath's experiments showed encouraging correlation with his theory. The measured phase distortion was approximately equal to zero as predicted by the theory. The dependence on k_x/k_y was found to be small.

A somewhat more refined analysis was presented by Liu (1973). Prescribing

- (1) continuity of normal velocity across the boundary
- (2) no slip at the boundary
- (3) continuity of stresses across the boundary and using the boundary layer approximation he found, for a fluid of depth H flowing over an infinitely deep bed, when the free surface was assumed to have the form

$$y_s = c e^{i2\pi\left(\frac{x}{L} - \frac{t}{T}\right)} \quad (36)$$

where

c = a coefficient

$i = (-1)^{1/2}$

y_s = vertical coordinate of the surface

that the pressure field in the fluid was given by

$$p = \left(\frac{i\rho c g}{C}\right) \left[\frac{2\pi}{TQ} \sinh \frac{2\pi(y+H)}{L} - i \cosh \frac{2\pi(y+H)}{L} \right] e^{i2\pi\left(\frac{x}{L} - \frac{t}{T}\right)} - \rho g y \quad (37)$$

where

$$C = \cosh \frac{2\pi H}{L} + \frac{2\pi}{TQ} \sinh \frac{2\pi H}{L}$$

$$Q = \frac{i2\pi}{nT} - \frac{\nu}{k}$$

H = depth of flow

n = porosity of sediment

k = permeability of sediment

ν = kinematic viscosity of fluid

$y = 0$ at the mean water surface and has its positive direction upwards

The pressure field in the bed was found to be given by

$$p = \left(\frac{\rho c g}{C}\right) e^{\frac{2\pi(y+H)}{L}} e^{i2\pi\left(\frac{x}{L} - \frac{t}{T}\right)} + \rho g h \quad (38)$$

The corresponding velocities in the bed are

$$u = \left(\frac{i2\pi c g}{LQC} \right) e^{\frac{2\pi(y+H)}{L}} e^{i2\pi\left(\frac{x}{L} - \frac{t}{T}\right)} \quad (39)$$

$$v = \left(\frac{2\pi c g}{QC} \right) e^{\frac{2\pi(y+H)}{L}} e^{i2\pi\left(\frac{x}{L} - \frac{t}{T}\right)} \quad (40)$$

The hydrodynamic effects of seepage on bed particles has been investigated by, among others, Watters and Rao (1971). Their conclusions were;

- (1) Seepage through the bed of an alluvial channel alters the flow configuration near the bed. In particular the effects of seepage are to modify the sublayer thickness of the channel bed, the particle boundary layer and the wake pattern behind the sediment particles. Further the effect of seepage is to modify the velocity profile near the channel bed. Consequently, the effects of seepage should be included in the stability criteria for a sediment particle.
- (2) Effluent seepage decreases drag regardless of the position of the sediment particle. The amount of influence depends on the shear velocity Reynolds number, the seepage Reynolds number and on the bed configuration.
- (3) The effect of seepage on the lift force acting on a plane bed particle is to increase or decrease it according to whether the seepage is upward or downward through the bed. However, the effects of seepage on particles above the bed are contrary to expectations. Effluent seepage reduces the lift on particles above the bed while the opposite is true for influent seepage.
- (4) Lift and drag forces are of comparable magnitudes and therefore the omission of lift forces in the stability criteria

for a bed particle seems to be unjustified.

- (5) Judging from the viewpoint of drag forces, effluent seepage inhibits the motion of a bed particle while influent seepage enhances its motion.
- (6) From the view point of lift forces, effluent seepage aids the motion of a plane bed particle but inhibits the motion of a particle above the general bed level. The opposite holds for influent seepage.
- (7) The variation of drag and lift coefficients for a plane bed particle with shear velocity Reynold number, Re^* , suggests that the functional relationship between them is strongly influenced by the conventional limits of hydrodynamic roughness, i.e.
 $Re^* < 70$ and $Re^* > 70$.

The fourth conclusion finds support in an investigation undertaken by Einstein and El Samni (1949). They measured the difference in mean static pressure in sediment beds at the level of the bottom of the top layer of sediment, and at the wall of the channel at the level of the top of the top layer of sediment. In these experiments the velocity was less than the critical and none of the sediment moved. Two sediment beds were used, one made of hemispheres of 0.225 ft diameter cemented to the bed in a hexagonal pattern, and the other gravel with mean size of approximately 0.225 ft. These measurements yielded a pressure difference, or lift pressure on the grains, of

$$\Delta p = 0.178 \frac{\rho u_0^2}{2} \quad (41)$$

in which u_0 is the velocity at a distance of $0.35 d_{35}$ above the theoretical bed. For the uniform hemispheres d_{35} was taken as the diameter, that is 0.225 ft. The theoretical bed level was defined as the position of the origin of y for which the measured velocity

profiles conformed to the assumed velocity profile

$$\frac{u}{u_*} = 2.5 \ln y/k_s + 8.5 \quad (42)$$

where

y = distance above the bed

k_s = equivalent sand roughness

This position was found to be below the tops of the uppermost grains, a distance equal to 0.2 times the diameter of the hemisphere bed and $0.2 d_{67}$ for the gravel bed. The values of

$$Re^* (= \frac{u_* d}{\nu}) \quad (43)$$

in the experiments were high, of approximately 50 000. Therefore the calculated lift pressure should be valid only for rough boundaries. It is interesting to compare the values of Δp with the boundary shear existing concurrently. This can be done by calculating the ratio of $\Delta p/\tau_0$ from the assumed velocity profile. This kind of calculation gives values of $\Delta p/\tau_0$ of about 2.5 if the standard deviation of the grain diameter is assumed to be about 1.4. Thus lift seems to be of considerable importance in entraining sediment.

Another study of the effects of seepage forces on the stability of a grain on a cohesionless bed was undertaken by Martin and Aral (1971). They found that seepage changes the slope at which the uppermost sediment layer reaches the threshold of instability. By equating the ratio of forces, normal and parallel to bed, to $\tan \phi$ (where ϕ is the angle of repose of the sediment) at the critical slope α_c , they obtained

$$\tan \phi = \frac{W \sin \alpha_c}{W \cos \alpha_c - F_s V} \quad (44)$$

where

W = representative weight of the bed particles

F_s = the seepage force per unit volume

V = volume of an uppermost bed particle and its surrounding fluid

The seepage force per unit volume of the top layer is defined as

$$F_s = c \frac{dp}{dy} \quad (45)$$

where

c = a coefficient relating the pressure gradient at the bed to the pressure gradient several layers below the bed.

$$\text{If } W = (\gamma_s - \gamma)V_s \quad (46)$$

where

V_s = the volume of the sediment particle

the coefficient c can be evaluated as

$$c = \frac{(\gamma_s - \gamma)(\sin \alpha_c - \cos \alpha_c \tan \phi)}{(1 + e) \frac{dp}{dy} \tan \phi} \quad (47)$$

where

e = the voids ratio

Martin and Aral also point out that the average bearing pressure, P_b , per unit volume is

$$P_b = \frac{dp}{dy} + \frac{\gamma_s - \gamma}{1 + e} \quad (48)$$

For vertically upward flow $P_b = 0$ implies a fluidized bed. Thus, when

$$\frac{\partial p}{\partial y} = - \frac{s_s - 1}{1 + e} \gamma \quad (49)$$

the seepage forces bring the bed particles to the threshold of movement.

For quartz-comprized particles in water for which $s_s \approx 2.65$ and for a bed of moderate to loose compactness $P_b = 0$ corresponds to

$$\frac{\partial p}{\partial y} = -\gamma \quad (50)$$

2.5 Present state of affairs

In spite of great efforts of a large number of researchers, in the field of local scour and sediment transport, the knowledge about the problem is far from satisfactory. In fact most of the intrinsic properties of the scour mechanism are more or less unknown. The opinions expressed by various researchers are widely diverging and the effects of different parameters are vividly debated.

Previous studies have shown that changes of bed elevation can have one or more of the following reasons;

- (1) general bed degradation or aggradation caused by changes of the hydraulic or hydrological properties of the water course.
- (2) degradation caused by contraction of the waterway or other changes of the geometry of the waterway.
- (3) changes of the cross section, as for instance a shift of the thalweg, or line of greatest depth, or similar effects of secondary currents.
- (4) local changes due to the disturbance of flow pattern caused by the presence of an obstruction such as a bridge pier or pipeline.

These processes are governed by different sets of parameters. Therefore it is necessary to separate the effects of each process and to treat them individually.

Shape	Length/width	Laursen and Toch (1956)	Tison (1940)	Chabert and Engel- dinger (1956)	Venkatadri et al (1965)
Circular Lenticular	1	1	1	1	1
	2	0.91			
	3	0.76			
	4		0.67	0.73	
	7		0.41		
Parabolic nose					
Triangular nose 60°					0.56
" " 90°					0.75
Elliptic	2	0.91			1.25
	3	0.83			
Ogival	4		0.86	0.92	
Joukowski	4			0.86	
	4.1		0.76		
Rectangular	2	1.11			
	4		1.40	1.11	
	6	1.11			

Table 2.3 Shape factors proposed by various authors

This separation of causes and effects has led to a great clarification of the problem. Another important distinction is the one between clear water scour and scour with continuous bed load movement, introduced by Laursen.

2.5.1 Location of maximum scour and form of scour hole

A matter, on which there is great agreement, is the shape and location of a scour hole near a bridge pier. For blunt nosed piers these holes are generally situated at the upstream pier nose and have the form of an inverted cone with the slope of the sides equal to the angle of repose of the sediment. Near streamlined piers, the location of maximum scour is shifted downstream, and the scour hole becomes much more shallow.

2.5.2 Influence of shape

Also on the effects of pier shape, most investigators express similar opinions. The general conclusion is; the blunter the pier the deeper the scour. The shape of the downstream end of the pier is not significant. A summary of coefficients found by different investigators is given in Table 2.3.

2.5.3 Influence of angle of attack

A related question is the influence of skew between flow direction and pier axis. The adverse effects of skew are recognized by all investigators but there is some variation in their opinions on the magnitude of these effects. Fig. 2.17.

2.5.4 Spacing of piers

The lateral spacing of bridge piers has been found to be insignificant. As long as the scour holes don't physically overlap, they develop as around a single pier. This implies that if the spacing is greater than $6B$, the scour holes can be treated individually.

If a pier is situated in the wake of another pier then the scour around the sheltered pier is considerably reduced.

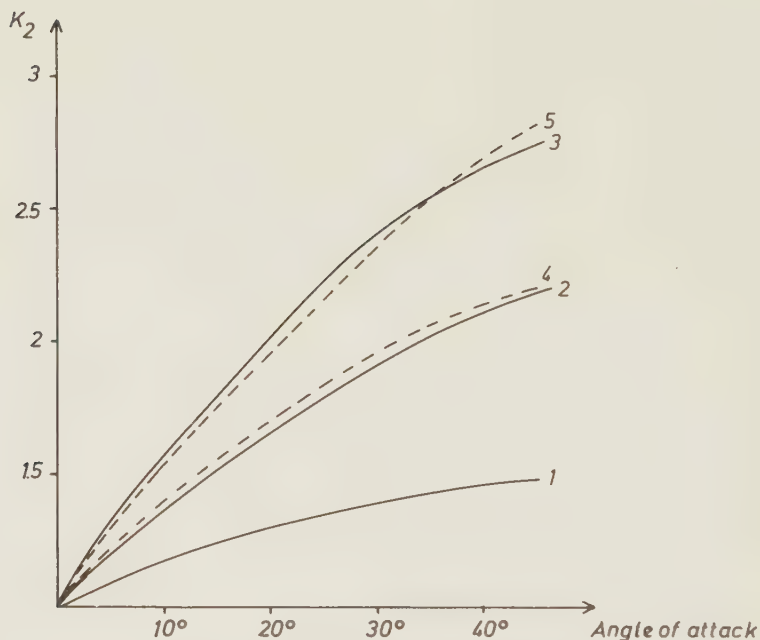


Fig. 2.17 Influence of skewness. (1) Laursen and Toch (1956) length/width 2:1, (2) Laursen and Toch 4:1, (3) Laursen and Toch 6:1, (4) Chabert and Engeldinger (1956) 4:1, (5) Shen et al (1966) 3:1

2.5.5 Influence of approach velocity

For clear water scour, the influence of flow velocity can be considered as well known. As the approach velocity increases, scour will commence close to the pier when the approach velocity amounts to a certain fraction, 0.5 for cylindrical piers, of the

critical velocity U_c . Then the depth scour increases linearly with velocity until the approach velocity is equal to U_c . Fig. 2.17.

When the approach velocity exceeds U_c , bed load movement is induced and the problem is transformed to the live bed problem. However, if U is only slightly greater than U_c the initiation of bed load motion may be hindered by for instance a vegetative cover. In such cases, the depth of scour may exceed the depth at U_c . Once the bed movement is fully established, the depth of scour is reduced somewhat below the depth at U_c . There is no longer any equilibrium scour depth but the depth of scour oscillates around a mean value. Fig. 2.19.

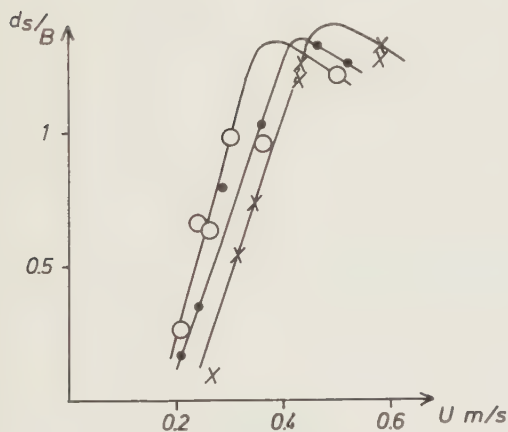


Fig. 2.18 Scour depth versus approach velocity for three different sands. (From Chitale [1962])

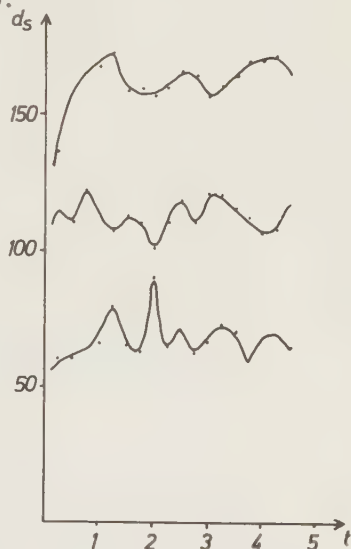


Fig. 2.19 Fluctuating scour depth (From Chabert and Engel-dinger [1956])

What happens to the scour depth as U increases considerably beyond U_c is not known exactly. Two additional factors may come into consideration;

- (1) as velocity increases an increasing fraction of the sediments transported goes into suspension

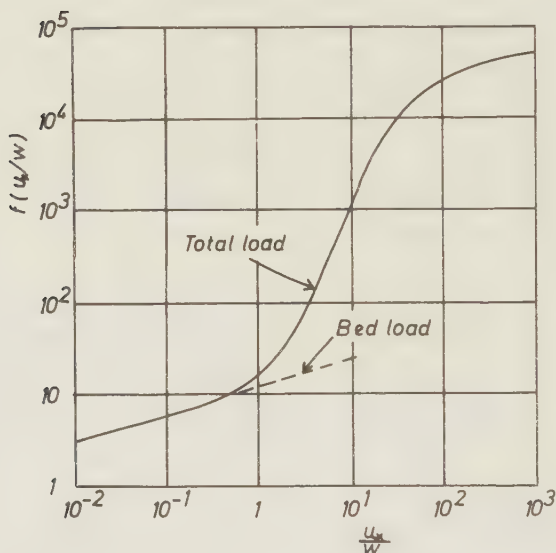


Fig. 2.20 Proportions between bed load and total sediment load according to Laursen (1958)
 w = particle fall velocity

This effect can severely affect the sediment balance of the scour hole and thus increase the maximum depth of scour. However, no generally agreed method of taking this effect into consideration has yet been presented. In most scour formulae the effect is excluded.

- (2) for velocities greater than U_c , a velocity change may induce a change of bed form. The effect of bed forms is to change the resistance to flow and thus the total shear. Further the bed form affects the proportions between effective shear

and form resistance. Thus a velocity change may lead to a change of bed form and thus to a shift of the sediment balance of the scour hole. No accepted method of estimating these effects exists.

2.5.6 Pier width

The opinions presented on the relationship between pier width and equilibrium scour depth are quite diverging. Roughly, these opinions can be summarized as; all other factors being kept constant, the depth of scour varies as B^α where B is the pier width and $0.5 \leq \alpha \leq 1$. For instance Breusers (1972) proposes $\alpha = 1$, Laursen's design curve corresponds to $\alpha \approx 0.7$, Larras (1963) suggests $\alpha = 0.75$ and Shen et al found $\alpha = 0.619$.

2.5.7 Sediment properties

For flow velocities lower than U_c , most researchers agree that the depth of scour is influenced by sediment properties. The general opinion is that the coarser the sediment the smaller the scour. When it comes to the importance of sediment gradation or porosity however, the standpoints become more indeterminate. However, for a specified mean diameter, a graded material seem to reduce the scour somewhat, compared to a uniform sediment, and the scour depth seems to increase with increasing porosity. Several suggestions about how to characterize sediment properties most efficiently have been presented, but none has been generally accepted.

For the live bed scour case, it is not clear if sediment properties have any influence at all on the magnitude of scour. According to Laursen, a change of sediment properties should affect the inflow and outflow sediment transport mechanisms in the same way, and therefore the equilibrium between these mechanisms should be unaffected, at least for non-cohesive sediments. This proposition has many supporters but it is undoubtedly con-

trated by some experimental results. Such results are found in the studies of for instance Chabert and Engeldinger (1956) or Nicollet and Ramette (1971).

However, there is also a vast amount of experimental results supporting Laursen's idea. Thus, it seems that the idea is generally correct but that it is subject to some, as yet, unidentified restrictions.

2.5.8 Influence of water depth

Here is another controversial question. For the case of continuous sediment transport most authors claim that below some limiting water depth, scour depth decreases with decreasing water depth but for water depths greater than the limiting depth the depth of scour becomes independent of water depth. The limiting depth is believed to be equal to one or two times the pier width.

On the other hand Laursen and most researchers influenced by the regime theory insist that for constant velocity, the depth of scour increases with increasing water depth.

For clear water scour an increase of water depth will increase U_c . Thus the depth of scour decreases as the depth of flow increases. Even for this case there is a limiting water depth, under which the depth of scour will decrease with decreasing water depth.

2.5.9 Lift and seepage forces

Such forces are seldom dealt with, despite clear experimental indications of their existence and their sometimes considerable magnitudes. Promising attempts have been made to evaluate the seepage forces, but none of the formulas derived is, as yet, directly applicable to local scour problems.

2.5.10 Sediment transport formulas

A large number of sediment transport formulas have been derived. However, most formulas have an enormous uncertainty. Computed values may differ by a factor of ten when compared to measured rates of sediment transport or when compared to values computed from another formula.

After an extensive test of sediment transport formulas Vanoni and others (1971) concluded that; sediment discharge formulas cannot be expected to give precise results. The Colby (1964), Tofaletti (1969) and Engelund-Hansen (1967) relations give consistently better agreement with data than any other formulas tested, and it seems that reasonable confidence can be placed in them. These are not conclusions but suggestions. The evidence available is insufficient to justify a more positive selection of relations. It is conceivable that, on the basis of evidence, workers will be justified in using other formulas besides the three singled out previously.

Calculations based on observed stream characteristics show much closer agreement than those based on average characteristics. Based on these results it is recommended that calculations with formulas be based on observed stream characteristics such as, slope, depth, velocity, bed material size and grading and temperature whenever possible. Also if reliable sediment discharge quantities are essential they should be based on suspended load samples and estimates of unmeasured discharge by the Modified Einstein (1950) method or by similar ones.

2.5.11 Scour protection

Many various devices have been tested. However, only two types of protections have been found really effective;

(1) one or more thinner piers upstream of the main pier

(2) a protective collar or layer at the pier footing.

Devices of the first kind may reduce the maximum scour depth by some 50 % but the efficacy is heavily dependent on the spacing and no rules giving the optimum spacing have been proposed.

The second method is better researched and there is a general agreement that such devices are effective provided they are placed at or, preferably, below the surrounding bed level. The lower such a device is placed, the lower is the required diameter. Optimum performance is expected if the top of the protection is situated half a pier width below the surrounding streambed.

2.5.12 Pipeline scour

Most studies indicate that the pipeline is safe if it is at least halfway embedded in the streambed or if the supporting bed is protected by a layer of gravel extending to the spring of the pipe.

No formulas defining critical conditions have been proposed.

3 PHILOSOPHY OF APPROACH

During the last fifty years a great deal of laboratory data, on the local scour process, has been collected. Considering this large amount of data, the uncertainty of existing scour formulas is embarrassingly great. One reason for this is the lack of relevant field data. Basically however, the greatest problem is the relative ignorance of the nature and properties of the underlying scour mechanisms.

Design and interpretation of laboratory experiments must be based on similarity criteria or dimensional analysis. If this is correctly done, similarity and dimensional analysis may be a powerful instrument, giving rise to criterial equations and providing a basis for criterial relations to their true and physical meaning. However, if the underlying concepts of the phenomena in question are too rudimentary there is a great risk for ending up with a confusing set of dimensionless criteria. These may not represent similarity and may have no physical interpretation.

The number of parameters judged to be of importance to the scour problem is large, much too large to be handled by purely empirical methods. The dimensional analysis may reduce the number of parameters somewhat, but it is still far too large to be handled empirically. The number of possible combinations of parameters is infinite and the selection of an effective set of parameters becomes an almost impossible task if the theoretical base is too vague. Furthermore, several of the parameters, for practical reasons, have to be neglected. For the estimation of the limits within which the thus reduced set of parameters gives a satisfactory measure of the process under consideration some knowledge about the character of this process is absolutely needed. That the present theoretical basis for dimensional analyses of scour problems is insufficient can be seen from the widely varying choices of principal governing parameters, present-

ted by various workers. Among the chosen parameters are such incompatible parameters as pier Froude number, pier Reynolds number and scour Euler number.

Admittedly the problem is very complex, involving such phenomena as separated flows and threedimensional boundary layers, and a final theoretical solution still seems to be very remote. However, there seems to be good possibilities of at least qualitative descriptions of some of the phenomena involved in local scour.

The major problem in connection with local scour is the insufficient knowledge of the flow pattern and velocity distribution induced by the presence of an obstruction. The change of flow properties near the obstacle changes the forces acting on the bed particles and usually leads to a change of bottom topography. This change of topography leads to a change of the flow field which changes the forces acting on the bed particles, and so on. These interaction phenomena are highly complex and little is known about them. Therefore it has been chosen to deal mainly with the flow field when the bed is kept plane. If the properties of this flow field could be established, a much sounder basis for estimation of the fluid forces acting on the bed particles should be created. For a general scour case, this is only a small step towards a characterization of scour hole developments but it is a large step towards assessment of necessary properties of a stable bed.

Thus the objectives of this study can be summarized as

- (1) to establish the flow pattern around an obstruction of simple geometry on a plane bed.
- (2) to study qualitatively the dependence of this flow pattern upon parameters such as approach velocity, depth of flow, shape of obstruction, and characteristic length of the obstruction.

(3) to study in what respect these flow patterns change the fluid action against the bed particles. The important aspects are believed to be;

- (a) change of boundary shear distribution.
- (b) changes of the lift or pressure forces.

4 ANALYSIS OF THE PROBLEM

Two kinds of flow fields are of interest in this study, the flow over a pipeline, lying on a plane bottom or partly embedded in it, and the flow around a vertical pier on a plane bed. In both cases, the flow is supposed to be at right angles to the longitudinal axis of the obstruction. Though the flow over a pipeline may not be perfectly two-dimensional, it will be treated as a two-dimensional flow in this study. The pier case however, represents the much more complicated and, as shall be seen, much more dangerous class of three-dimensional flows.

4.1 Two-dimensional flow over pipeline

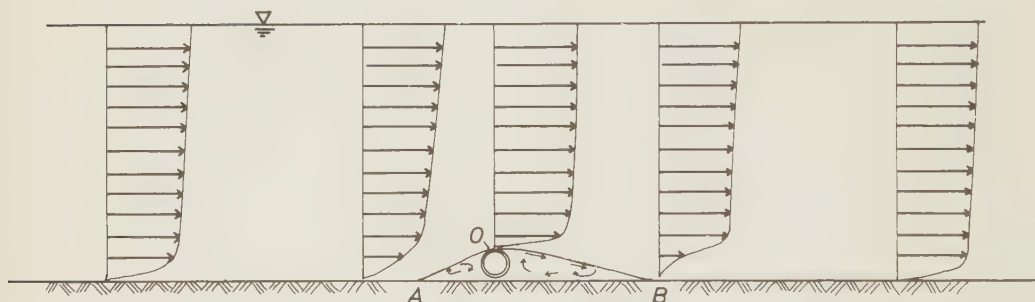


Fig. 4.1 Schematic representation of the flow over a pipeline

Consider the flow over the pipe sketched in Fig. 4.1. The approach flow can be considered to have a vertical velocity variation corresponding to the boundary over a rough plate. Upstream of the pipe, a pressure field is induced by the presence of the pipe. The effect of this 'secondary' pressure field is to decrease the approach flow velocities near the bed. At some point, A, the adverse pressure gradient becomes strong enough to cause separation of the approach flow. The position of the point A is not constant but varies with the inherent velocity and pressure fluctuations of the flow. Somewhere along the circumference of the pipe, the flow reattaches to the boundary but in the vicinity of point O the flow is again separated.

Downstream of the pipe the flow reattaches to the boundary again at some point B. As for the separation point, the position of the point of reattachment is fluctuating about some average position, given by the average flow conditions.

Under the separated flows two secondary flow systems are developed, both characterized by rotating motions leading to flow directions which, near the bed, are opposite to the approach flow direction.

As the boundary shear stress is proportional to the velocity gradient at the foot of the velocity profile, the effects of the pipe on the mean boundary shear can be concluded. A well-known criteria for a point of separation is that the velocity gradient in a direction normal to the surface disappears. Thus, upstream of the point A the boundary shear, τ_0 , of the approach flow decreases monotonously until it attains the value zero at A. Downstream of the point B, the reattachment profile is continuously redistributed until the profile is again similar to the undisturbed velocity profile of the approach flow. Between the points A and B the primary flow is nowhere in contact with the bed. Thus, the influence of the pipeline on the primary flow is to diminish the mean boundary shear outside the region A-B.

For the clear water scour case, there is reason to believe that the primary flow does not at all cause any sediment movement. For the case of continuous bed load motion however, the diminishing shear stress upstream of point A may lead to a decrease of the bed load transport rate and may thus lead to an accretion in the vicinity of point A. This accretion cannot go on indefinitely and eventually the continuity of the bed load transport is re-established. While deposition is occurring on the upstream side of the pipeline, the reattaching flow at B may carry less sediment than corresponds to its capacity. Thus the primary flow may lead to limited erosion at or downstream of point B. However, this area is at too large a distance from the pipe for this possible scour to affect the stability of the pipe foundation

Thus, the reasons for undermining of pipelines have to be sought in the secondary flows. Undoubtedly there is a pressure increase upstream of the pipe and a pressure decrease downstream. If the length of the pipe wall in contact with the bed is small, it is conceivable that pressure gradients may cause piping or severely reduce the bed resistance to other fluid forces. This seems to be the most important mechanism in pipeline scour.

Because the separation surface upstream diverges from the bed, the mean boundary shear of the secondary flow under this surface can be presumed to be smaller than the boundary shear of the oncoming flow. However, turbulence may cause fluctuations of considerable amplitude of as well shear stresses as pressure. Furthermore influence of turbulence on settling must not be overlooked.

The secondary flow on the downstream side can be presumed to have similar properties, the most important difference being that, with reservation for possible tertiary flow, the direction of the flow close to the bed is directed towards the pipe.

4.2 Three-dimensional flow about a cylinder

Let a vertical cylinder be placed on a plane horizontal bed in a wide stream. Far from the cylinder, the velocity distribution of the stream flow may be assumed to be in accordance with the ordinary boundary layer velocity profile.

For near two-dimensional, that is for uniform oncoming flow, there would be a stagnation line somewhere on the upstream side of the cylinder. The exact location of this line would depend on the cross-section of the cylinder. Furthermore the streamlines of the approach flow would be deflected around the cylinder but would still be in horizontal parallel planes. But if the approach flow has a velocity gradient, then the pressure along the vertical stagnation line would vary according to the stagnation pressure of the different fluid layers. Also along all other verticals within the fluid affected by the cylinder, there would be a

vertical pressure gradient. These pressure gradients give rise to vertical secondary flows. On the upstream side these secondary flows are downward, at least in the lower layers of the fluid. Thus, the streamlines are no longer in any horizontal planes but converge towards the bed. This leads to an increase of the fluid velocities in the lower layers and thus to an increase of mean boundary shear. This is considered to be a very important mechanism of local scour around bridge piers.

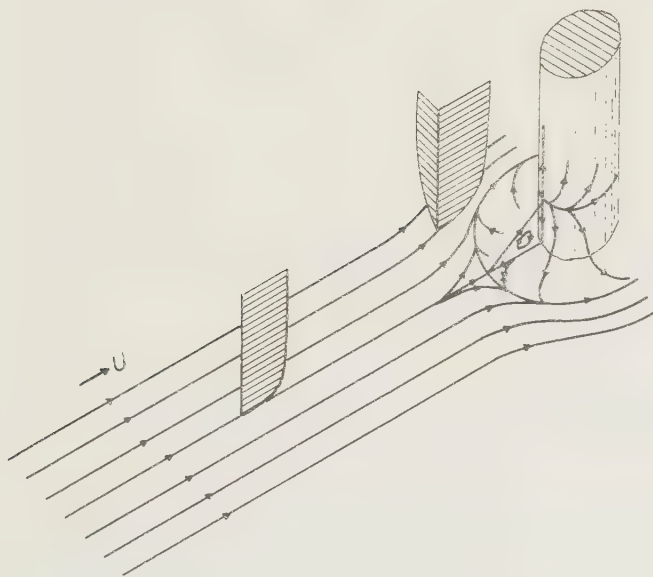


Fig. 4.2 Schematic picture of flow around a bridge pier

If the cylinder is blunt, the pressure field will be strong enough to cause a downward flow on the front face of the cylinder. This leads to a separation of the boundary layer which

rolls up ahead of the pier base to form a vortex, known as the horse-shoe vortex system.

In its simplest form the horse-shoe vortex system consists of one large vortex with a small counter vortex, Fig. 4.3.

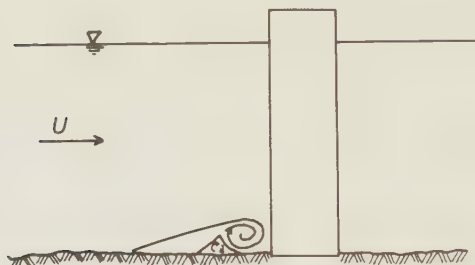


Fig. 4.3 Horse-shoe vortex with small secondary vortex as observed by Schwind (1962)

It also exists in higher regimes containing multiple vortices. These seem to be unstable and are near periodically shed. The process bears some resemblance to vortex shedding behind cylinders in two-dimensional flow.

Most workers agree that the horse-shoe vortex system plays the most important part in the local scour process around bridge piers. However, the exact mechanism of the system is not known.

The vortex is believed to act more through its influence on the properties of the approach flow than through its own scouring potential. The downward secondary flow affects the vorticity convection and thus tends to concentrate the vorticity towards the bed. This effect may considerably increase the boundary shear.

For the continuous sediment transport case two more effects have to be considered. The vortex system deflects the neighbouring

streamlines and thus effectively hinders any bed load transport into the area covered by the vortex. If the scour hole is bigger than the vortex however, bed load will be fed into the hole and then slide down into areas covered by the vortex. But even in this case the deflection of the streamlines will somewhat reduce the sediment inflow. Thus the vortex would induce scour around the pier even if the strength of the flow in the vortex were equal to the strength of the approach flow.

A similar effect is caused by the diving motion in the vortex. This means that the water brought into contact with the bed under the vortex comes from higher layers of the approach flow and thus carries far less sediment than corresponds to its capacity. The tendency of the flow to compensate for this may thus increase the scour.

Behind the cylinder there will be a zone of highly turbulent wake flow. The properties of such flows are known only for two-dimensional motion with very low values of Reynolds number. Though the sediment transport rate in this zone may be small, observations generally show the bed sediment to be in a forceful but random motion. The vortex flow is enclosed within the free shear layers extending downstream from the lines of separation on the cylinder. Thus the wake flow is highly dependent on the vortex shedding from the cylinder. The free shear layers separate regions of different pressure, the pressure in the wake being less than the pressure in the surrounding fluid. This may lead to destabilizing seepage under the shear layer. More important is maybe the shifting position, due to shedding, of the vortex sheet. The pressure at a given point on the bed may vary considerably depending on whether the point is at the moment on the inside or on the outside of the vortex sheet.

The pressure gradients induced by the presence of the pier may be considerable. For instance at a rectangular pier, the pressure against the front side, near the upstream corner, is close to the full stagnation pressure. Just around this corner, the pressure attains its minimum value and it is conceivable that the thereby generated pressure gradient may be important enough to severely

affect the stability of the bed sediment.

Thus the flow and pressure fields induced by the cylinder affect the sediment equilibrium in several ways. Separation of the different causes and effects seems to be the most fruitful approach. Therefore the influence of the horse-shoe vortex system upon the transport capacity out of the region covered by the vortex will be dealt with primarily.

The interest will be centered on forces under the vortex when the bed is stable and flat. This corresponds to a study of the forces acting on a resistant scour protection.

Most scour studies have dealt with limiting equilibrium scour depth. However, to be able to deal with vortex properties over a deformable bed one has to consider two additional processes, about which the present knowledge is utterly limited. These processes are the influence of the vortex on the bed shape and the influence of the bed shape on the vortex properties. These processes are by no means considered as less important but it is felt that the combined effects of these processes cannot be satisfactorily analyzed until the basic properties of each process are better known.

5 ANALYTIC DESCRIPTIONS OF THE FLOWS

5.1 Introductory remarks

The most important objective is to describe the mechanics of the boundary layers. This is most effectively done by studying the flow in terms of arguments considering vorticity. It is true that the phenomena could also be discussed in terms of momentum changes due to convection, viscous diffusion, and the action of pressure gradients. But, to explain convincingly the existence of boundary layers, and, also, to show what consequences of flow separation may be expected, arguments concerning vorticity are needed. Furthermore, such arguments are needed to explain why the flow outside the boundary layer and wake should be irrotational.

Thus, although the local behaviour in a boundary layer can be convincingly explained by momentum considerations, vorticity considerations are necessary to place the boundary layer correctly in the flow as a whole. It can be shown that such considerations illuminate the detailed development of the boundary layer just as clearly as do momentum considerations.

The vorticity vector, $\omega = (\omega_x, \omega_y, \omega_z)$ is defined as the curl of the velocity vector $V = (u, v, w)$

$$\omega = \text{curl } V = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \quad (1)$$

It is easily seen that, like V , ω satisfies the solenoidality or continuity condition

$$\text{div } \omega = \frac{\partial \omega_x}{\partial x} + \frac{\partial \omega_y}{\partial y} + \frac{\partial \omega_z}{\partial z} = 0 \quad (2)$$

Geometrically, this means that the magnitude of the vorticity varies along any elementary vortex tube inversely as the cross-sectional area of the tube.

If the solenoidality condition is combined with Helmholtz's theorem, stating that vortex lines move with the fluid, a law for the behaviour of vorticity in an ideal fluid is obtained. This rule can be given a geometrical interpretation; if one imagines the fluid flow at any instant as including tiny arrow-shaped elements of fluid, each pointing in the direction of the local vorticity and with a length proportional to the magnitude of the vorticity, then these individual arrow shaped elements will move so that at all subsequent times they give the same information with the same accuracy.

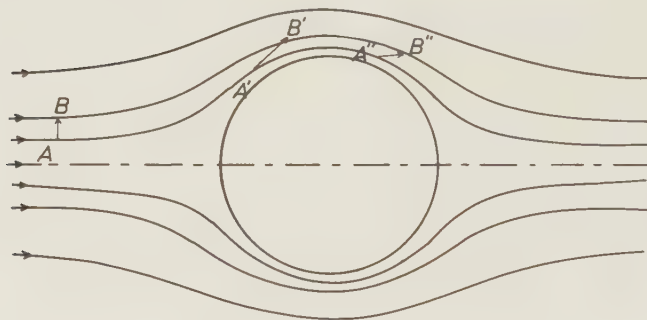


Fig. 5.1 Deformation of small arrow-shaped fluid element by convection

The principle is illustrated in Fig. 5.1 which shows the streamlines for an ideal potential flow about a cylinder. Suppose, that far upstream of the cylinder, the flow contains vorticity, whose direction is perpendicular to the flow. Thus the undisturbed vorticity can be represented by the arrow A-B, the length of which is proportional to the strength of the vorticity. A certain time later, the fluid of point A has been convected to the new position A' and B has shifted to B'. According to the above rule, the arrow A'-B' represents the vorticity at

the new location. Thus it is seen that pure convection has changed the direction of vorticity as well as the magnitude.

In a real fluid, the angular momentum of an element changes as a result of tangential stresses proportional to the viscosity. This leads to an additional term in the rate of change of vorticity, which can be interpreted as due to diffusion with diffusivity ν . Each term in the expression for ω is proportional to a momentum gradient, whose change due to viscosity must correspond to diffusion with this diffusivity, simply because it is so for momentum itself. Thus for a real fluid we have;

$$\frac{d\omega}{dt} = (\omega \nabla) V + \nu \nabla^2 \omega \quad (3)$$

The inertial rate of change of ω for a fluid particle, represented by the turning and stretching of the vortex lines, must therefore be supplemented by a diffusion term $\nu \nabla^2 \omega$. The corresponding contribution of diffusion to the rate of change of total vorticity in any volume of fluid is equivalent to a flow, across unit area of its surface, at a rate ν times the gradient of ω along the outward normal.

This important idea of vorticity flow across a surface cannot be viewed as diffusion of angular momentum because the vorticity is proportional to the angular momentum of a spherical particle about its mass centre, and when it diffuses from one particle to another the relevant angular momentum is about quite a different point. The correct view relates diffusion of vorticity to that of momentum, through that of momentum gradient.

It is well known, that in an ideal fluid, a particle of fluid with zero vorticity will continue to have zero vorticity. In a viscous fluid this doesn't hold since diffusion from nearby particles may occur. However, diffusion in itself doesn't create vorticity and one may reasonably ask; how is vorticity

imparted to a fluid, all of which lacks it initially. The answer is that solid boundaries create vorticity and thus act as a distributed source of vorticity.

The tangential vorticity source strength has a simple relation to pressure gradient, at least for flows over plane stationary surfaces. If the surface is taken as $z = 0$ the flow of x-vorticity out of it is

$$-v \frac{\partial \omega_x}{\partial z} = -v \frac{\partial}{\partial z} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) = v \frac{\partial^2 v}{\partial z^2} = v \nabla^2 v = \frac{1}{\rho} \frac{\partial p}{\partial y} \quad (4)$$

since u, v, w are zero on $z = 0$, and because transfer of momentum by convection is absent at a solid surface, so that transfer by diffusion must exactly balance transfer by pressure gradient. In general it is required that the solenoidality condition is maintained. This, indeed, relates the normal vorticity source strength directly to the surface distribution of tangential vorticity by the equation

$$-v \frac{\partial \omega_z}{\partial z} = v \left(\frac{\partial \omega_x}{\partial x} + \frac{\partial \omega_y}{\partial y} \right) \quad (5)$$

The vorticity, which is produced at the solid boundaries, and carried away from them by diffusion and convection, determines the entire flow, which in turn controls the production of vorticity.

The proof of this result is given in two parts. First a velocity field V_0 , which has the given vorticity distribution and tends to zero at infinity is established. This is given by the Biot-Savart law

$$V_0 = \int_{V_f} \frac{\omega \times r}{4\pi |r|^3} dv \quad (6)$$

where

the integration is over the whole vorticity field

r = the position vector relative to the volume element dv

In general V_0 does not satisfy the boundary condition of zero normal velocity. This can be corrected for by superposition of a velocity field V_1 . If V is a flow field which has the given vorticity distribution and satisfies the boundary conditions then $V = V_0 + V_1$. Thus, V_1 must be irrotational since $\text{curl } V_1 = \text{curl } (V - V_0) = \text{curl } V - \text{curl } V_0 = \omega - \omega$. Hence V_1 is equal to the gradient of some potential ϕ . Thus on the body surface

$$\frac{\partial \phi}{\partial n} = V_n - V_{0n} = V_{\omega n} - V_{0n} \quad (7)$$

which is fixed, and further $\text{grad } \phi$ tends toward zero at infinity.

Now, Lamb (1932) has shown that, if a body moves through unlimited fluid, which is at rest far from the body, then one and only one flow, with zero vorticity everywhere, satisfies the boundary condition of zero normal velocity at the solid surface. Thus, just one ϕ , except for an arbitrary constant, satisfies these conditions.

This addition, $\text{grad } \phi$, to the Biot-Savart velocity field is sometimes described as the image vorticity, since, for simple shapes of boundary, it can easily be related, by the same Biot-Savart formula, to a distribution of virtual or image vorticity within the body, Lighthill (1956b).

Thus, any flow development is in principle computable by studying the diffusion of vortex lines, and their stretching and convection by the associated flow, while supposing normal and tangential vorticity to appear continuously at the solid surfaces in just such measure as is required to maintain respectively the solenoidality of the vorticity field and the no-slip condition.

For a step-by-step computation of vorticity development, a method of progressing from one vorticity distribution to the vorticity distribution of the next step is needed. The rate of change of vorticity is that generated by the flow field in stretching and convecting the vortex lines plus that produced by viscous diffusion. To complete the knowledge of the vorticity

field at the later instant, the rate of production of vorticity at the solid boundaries must be determined. To find this, one can assume it to be zero at the first step and proceed to the next stage of the problem; to determine the velocity of the next step. This requires only a knowledge of the original vorticity distribution and the normal velocities at the solid boundaries. The resulting tangential velocity may not satisfy the no-slip condition. This can be corrected for by assuming that exactly enough tangential vorticity has been created at the solid surfaces to give a velocity field, which combined with that previously given gives zero slip.

Some of the above conclusions will have to be modified if the effects of turbulence are taken into consideration. However, this is more readily done after a full discussion of the mechanics of laminar flow.

If the streamlines close to a surface are closely parallel to it, convection prevents the region of vorticity near the surface to grow beyond a thickness of order

$$[\nu l/U_\infty]^{1/2} \quad (8)$$

where

l = the distance from the point of generation of the vorticity

U_∞ = the velocity at the outer edge of the vorticity region

This is because the vorticity in the outer parts of the boundary layer is convected at a speed which is close to that of the irrotational flow outside the layer, and therefore of order U_∞ ; hence the time since the generation of a vortex element is at most of order l/U_∞ , during which it can have diffused a distance at most of order $[\nu l/U_\infty]^{1/2}$ away from the surface. This rule is generally correct but has important exceptions.

An especially interesting class of flows are those where the boundary layer thickness remains small over the whole surface. Such flows are called thin wake flows and have the valuable property that they may be approximately computed in three steps, of which just the first two or even the first one only may often be regarded as sufficient.

The first step is to determine the irrotational flow outside the boundary layer and wake by ignoring the former and replacing the latter by a vortex-sheet, the total vorticity per unit area being unchanged.

Next step is to determine the detailed vorticity distribution within the boundary layer from the knowledge of the external flow.

Finally the modifications of the irrotational flow due to the calculated properties of the boundary layer are determined.

5.2 The pipeline case

If the pipeline is at right angles to the approach flow, the flow pattern over the pipe may be considered as essentially two-dimensional.

The potential ideal fluid motion over a more or less embedded pipeline is exactly analogous to the two-dimensional potential flow around a cylinder if the bed is supposed to correspond to the line of symmetry. Same equivalent cylinder cross-sections are shown in Fig. 5.2

The stream and potential functions Ψ and ϕ for flow around cylinders of this type are given by for instance Milne-Thomson (1962).

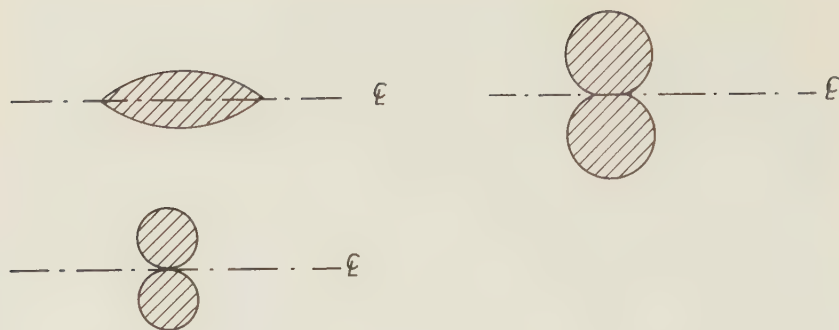


Fig. 5.2 Analogue cylinder shapes for various degrees of embedding of pipeline

For the sake of convenience, the flow over a pipe, half-way embedded, will be considered. The analog cylinder is the circular cylinder, Fig. 5.2, and the flow is given by the well-known complex potential

$$\Omega = \phi + i\psi = U(x + iy + \frac{a^2}{x + iy}) \quad (9)$$

where

U = the velocity at infinity

a = the radius of the cylinder

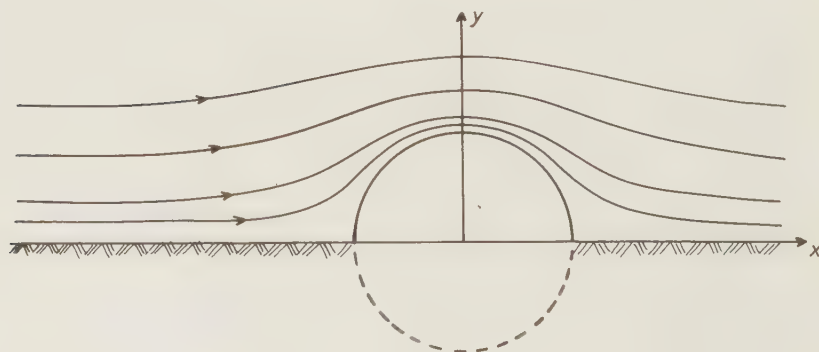


Fig. 5.3 Streamlines over half-way embedded pipeline.

The greatest simplifying, feature of two-dimensional flows around this kind of obstruction is that all vortex lines are parallel to the generators of the cylinder at all times, because physically, any spin must be about this direction. Convection of vortex lines cannot therefore stretch or rotate them, so that values of ω are convected without alteration except that due to diffusion.

The matter of interest is therefore how the effects of diffusion are balanced by effects of convection. To find out about that one has to ask: how parallel to the surface are the neighbouring streamlines? Along any such streamline

$$\frac{1}{2}\omega_w z^2 = \epsilon \quad (10)$$

where

ω_w = the vorticity at the wall

z = the distance of the streamline from the wall

ϵ = a constant volume flow per unit span between the streamline and the surface

Hence, the distance z from the surface varies as $\omega_w^{-\frac{1}{2}}$, which implies only a moderate variation except where ω_w approaches zero. However, according to Rosenhead (1963), this equation loses accuracy as ω_w approaches zero, and has to be supplemented with the z^3 - term in the expansion of the volume flow in powers of z . For such cases the equation becomes

$$\frac{1}{2}\omega_w z^2 + \frac{1}{6}\left(\frac{1}{\mu}\frac{\partial p_w}{\partial x} - \kappa\omega_w\right)z^3 = \epsilon \quad (11)$$

where

p_w = the pressure at the surface

κ = the curvature of the surface

Still solutions for z remain small, and close to $(2\epsilon/\omega_w)^{1/2}$ except at points of separation or attachment where they cease to be small.

To proceed further, it is necessary to consider the actual vorticity source strength per unit area of surface, and to use an approximate relationship between vorticity and velocity in the boundary layer. In line with normal practice this relationship can be represented by the boundary layer approximation introduced by Prandtl

$$\omega = \partial u / \partial z \quad (12)$$

Now U_∞ , defined as the total vorticity per unit area of boundary layer at a given point of the surface, can be written as

$$U_\infty = \int_0^\delta \omega dz \quad (13)$$

where

δ = the value of z at the outer edge of the boundary layer

Hence, on the boundary layer approximation U_∞ is the velocity at $z = \delta$. For this reason it may be called the external velocity.

To the same approximation, the rate of convection of vorticity, per unit span, past a given point of the surface is

$$\int_0^\delta u \omega dz = \int_0^\delta u (\partial u / \partial z) dz = \frac{1}{2} U_\infty^2 \quad (14)$$

which shows that the mean velocity of convection is $\frac{1}{2} U_\infty$. The variation of this vorticity-flow rate with x can in steady flow be due only to new vorticity, whose rate of creation at the boundary is therefore

$$d\left(\frac{1}{2} U_\infty^2\right)/dx = U_\infty \frac{\partial U_\infty}{\partial x} \quad (15)$$

per unit area. Where $\frac{\partial U_{\infty}}{\partial x}$ is negative, the divergence of streamlines in the boundary layer supplements diffusion in producing thickening. Just before separation the thickening becomes particularly marked, and the boundary layer approximations then become inadequate. Nevertheless they have led to reasonable agreement with experiments in predictions of the point of separation and the flow upstream of this point.

Following the principles outlined above, one finds the irrotational velocity distribution over a half-way embedded pipe-line to be given by

$$u = U_{\infty} \left(1 - \frac{a^2(x^2 - y^2)}{(x^2 + y^2)^2} \right) \quad (16)$$

$$v = - U_{\infty} \frac{2a^2xy}{(x^2 + y^2)^2} \quad (17)$$

Now, if the no-slip condition is satisfied by the introduction of a vortex sheet along the boundary, the total vorticity per unit area becomes

$$U_{\infty} \left(1 - \frac{a^2}{x^2} \right) \quad (18)$$

along the bed, and

$$2U_{\infty} \frac{y}{x} \quad (19)$$

along the surface of the pipe.

At the bed, $y = 0$ and thus

$$\frac{\partial U_{\infty}}{\partial x} = 2U_{\infty} \frac{a^2}{x^3} \quad (20)$$

From this it can be concluded, that the vorticity source strength diminishes as the flow approaches the pipe, and becomes zero just in front of the pipe.

Thus closely upstream of the pipe, the vorticity source strength is low and convection is away from the surface. Therefore, the boundary layer thickens rapidly and the flow separates.

It can easily be concluded that the boundary shear of the on-coming flow decreases monotonously to become zero at the point of separation.

The flow under the surface of separation is extremely complicated and not amenable to theoretical analysis. However, some qualitative conclusions considering the mean boundary shear can be drawn.

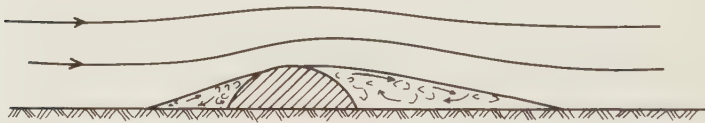


Fig. 5.4 Zones of separation near pipeline

The flow receives its energy from vorticity diffusing from the surface of separation. The flow thus driven produces vorticity of opposite sign at the bed, and this vorticity will diffuse to cancel some of that diffusing from above.

Close to the point of separation the vorticity at the bed is zero. However, along the surface of separation, production of vorticity is increased and furthermore convection tends to concentrate vorticity, at the surface. These conditions prevail all the way to the top of the pipe, where the strength of vorticity reaches its maximum, of order U_∞ . Thus, where the diffusing distance, i.e. the distance between the surface of separation

and the bed is short, the rate of vorticity production and transfer is small. Where the rate of vorticity production is high, the diffusing distance is considerable. Thus, the mean boundary shear between the point of separation and the pipe can be safely considered to be well below the shear stress of the oncoming flow.

On the downstream side of the crest of the pipe, the streamlines diverge. This is most pronounced close to the surface of the pipe and leads to a second separation of the flow. A process, which could be described as the reversal of the upstream separation behaviour, is initiated. However, in this case convection is directed away from the surface of separation and therefore the length of the separation surface is greater on the downstream side. Experimental investigations have shown this length to be about six times the height of the obstruction.

The mean bed shear should be even lower than at the upstream side. Furthermore, the direction of the flow close to the bed is directed towards the pipe. Thus the mean boundary shear in the downstream wake can hardly lead to undermining of the pipeline.

At the point of reattachment, the mean boundary shear is zero. Downstream of this point we have

$$\frac{\partial U_{\infty}}{\partial x} = 2U_{\infty} \frac{a^2}{x^3} > 0 \quad (21)$$

and thus the mean boundary shear increases monotonously until it approaches the boundary shear of the undisturbed flow far downstream.

Thus the presence of the pipe does not in any point of the boundary increase the mean shear stress above the mean shear stress of the undisturbed flow. However, turbulent fluctuations of shear stresses in this kind of flow may be important enough

to increase the scouring capacity of the flow. To this question there is no way to make any analytic estimate of the answer. Resort has to be taken to experimental results.

Most likely, the undermining of pipelines is mainly caused by pressure variations generated by the pipe. On the upstream side of the pipe, the mean pressure can be considered to correspond to the stagnation pressure of the approach flow at some level between the bed and the crest of the pipe. Downstream of the pipe, the pressure corresponds to the pressure immediately downstream of the point of separation. The potential flow solution gives a pressure difference between the maximum and the minimum pressure of

$$\Delta p = 4 \frac{\rho u^2}{2} \quad (22)$$

but for real fluids, the pressure difference is somewhat less. Still it is conceivable that the pressure gradient in the bed under the pipe is strong enough to generate a non-negligible seepage. When the length of pipe perimeter in contact with the bed is small the pressure gradient may become strong enough to wash out the sand under the pipe.

Even if the pressure gradient is not strong enough to wash out the sand, the seepage created by it will affect the stability of the top layer of sand which may thus be scoured away by shear stresses far less than the critical stress under normal conditions.

On top of the mean pressure difference, there is a fluctuating pressure difference caused by turbulent fluctuations of the flow. The magnitude of these fluctuations is hard to estimate but they may be important. The pressure fluctuations are proportional to the square of the velocity fluctuations. However, as neither velocity nor pressure fluctuations are yet analytically

derivable, direct measurements will have to be done to evaluate their amplitudes.

5.3 Flow around cylinders

The quantitative theory of general three-dimensional flow in boundary layers is pitifully underdeveloped. Accordingly there are no methods to give anything but a crude qualitative picture of the factors influencing the vorticity distribution, both through the layer and over the surface.

Streamlines very near to the surface lie closely along the skin-friction lines or neighbouring streamlines, however, their distance z to the surface varies, not as $\omega_w^{-1} z$ as in the two-dimensional case, but as

$$(\omega_w h)^{-\frac{1}{2}} \quad (23)$$

where

h = the distance between two adjacent skin-friction lines

This is because in a streamtube, whose base is the portion of the surface between the two skin-friction lines and whose, variable, height is z , the volume flow along the streamtube, ϵ , is given by

$$\frac{1}{2} \omega_w z^2 h = \epsilon \quad (24)$$

It follows that streamlines can increase greatly their distance from the surface, not only where ω_w becomes very small but also where h does, that is, where skin-friction lines run close together. These alternative mechanisms need to be borne continuously in mind. Thus, even if stretching of vortex lines is relatively immaterial, their rotation may be of first importance.

5.3.1 The secondary flow upstream of a cylinder

The full problem of three-dimensional flow around a cylinder is far too complex for an analytical solution but interesting approximate methods for analysis of this type of flows have been presented.

Hawthorne (1954) used a small perturbation technique to evaluate the energy of the secondary flow about struts and airfoils when the approaching velocity varies in the spanwise direction. The theory requires that the fluid is inviscid and incompressible and that the flow is a small disturbance of the two-dimensional potential pattern appropriate to the profile. It was found that the shape of the profile at the nose influences the size of the disturbances appreciably. A round nose causes such a large disturbance that the assumptions of the theory are invalidated. For wedge-shaped or cusped profiles, which cause smaller disturbances, the theory gives interesting results.

Hawthorne examined the flow around a bicusped profile and computed the energy of the secondary flow normal to the approach flow for one type of initial velocity distribution. This energy was found to vary as the fourth power of the thickness of the strut.

Calculations were also made for other thin airfoils. The major effects predicted by the theory were confirmed by comparison with experiments.

Though Hawthorne's theory is interesting, it is less suited for calculations of flow properties around blunt-nosed cylinder shapes.

An approach, which seems better suited for blunt cylinder shapes was presented by Lighthill (1956 a). He used the simple shear approximation, i.e. the assumption that the vortex lines are

convected and deformed by the primary flow alone, to evaluate the secondary flow around an infinitely long cylinder of arbitrary cross-section for the case that the oncoming flow has a small and constant velocity gradient along the axis of the cylinder.

To compute the stretching and rotation of vortex lines, Light-hill used the findings of Darwin (1954), who found that the deformation or 'drift' as he called it, of fluid planes can be described in terms of a time function. Darwin defined this time function $t(x,y,z)$ as the time since the fluid particle passed a plane normal to the undisturbed flow. However, Light-hill found it more convenient to define it as the time at which the particle would have passed a plane normal to the undisturbed flow, had the flow remained undisturbed. That is, if the relevant plane is taken as $x' = 0$;

$$dt = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \quad \text{with } t - \frac{x}{u} \rightarrow 0 \text{ as } x \rightarrow \infty \quad (25)$$

where

u, v, w = the velocity components of the x, y, z directions respectively and where, far upstream

$$u = U$$

$$v = w = 0$$

The advantage of this definition over Darwin's is that, far upstream, material planes at right angles to the stream are planes $t = \text{constant}$; hence the surfaces into which these are deformed, in which there is particular interest, are all given by $t = \text{constant}$.

It is easy, in terms of this drift function, t , to determine how vortex lines, initially at right angles to the stream, are deformed and stretched by the irrotational flow about the body. (The simple shear approximation is that the vortex lines

are considered weak enough for the additional stretching due to their presence to be neglected.) A vector element joining a pair of points on neighbouring streamlines will always join points on these two streamlines, and they will always be points with the same value of t . The vector separation of these two points, divided by the scalar separation of the original pair, gives the direction and magnitude of the vorticity divided by its upstream magnitude.

Lighthill's theory is developed for flow past an obstacle in which the velocity field far upstream is a parallel flow

$$u = V(y,z) , \quad v = w = 0 \quad (26)$$

The obstacle is supposed to be near the axis $y = z = 0$ and $V(y,z)$ is supposed to vary only a little from its value $V(0,0) = U$ over a range of variation of y and z large compared with the size of the obstacle. If the gradient of V also varies little over this range then a suitable choice of axes gives

$$u = U + Az , \quad v = w = 0 \quad (27)$$

far upstream, for some rate of shear A .

In all cases, the irrotational flow about the obstacle with uniform upstream velocity U is called the primary flow.

The streamlines of the primary flow are represented by equations of the form

$$y = y(x, y_0, z_0) , \quad z = z(x, y_0, z_0) \quad (28)$$

where

$$\begin{aligned} y_0 &= \lim_{y \rightarrow -\infty} y \\ z_0 &= \lim_{z \rightarrow -\infty} z \end{aligned} \quad (29)$$

The solution for the additional variable t is

$$t(x, y_0, z_0) = \frac{x}{U} + \int_{-\infty}^x \left(\frac{1}{u(x, y_0, z_0)} - \frac{1}{U} \right) dx \quad (30)$$

where

$u(x, y_0, z_0)$ signifies the value of the velocity component u on the streamline given by (28)

In case (27) the upstream vorticity field is given by

$$\omega_x = \omega_z = 0, \quad \omega_y = -A \quad (31)$$

where

$\omega_x, \omega_y, \omega_z$ = the vorticity components along the axis x, y, z respectively

An element of vortex line is deformed by the primary flow but always remains on a surface $t = \text{constant}$. Thus, the vorticity is given by

$$\omega_x = Au \frac{\partial t}{\partial y_0}, \quad \omega_y = A \left(v \frac{\partial t}{\partial y_0} - \frac{\partial y}{\partial y_0} \right), \quad \omega_z = A \left(w \frac{\partial t}{\partial y_0} - \frac{\partial z}{\partial y_0} \right) \quad (32)$$

The secondary flow associated with the secondary vorticity field is calculated in three parts.

- (1) a part, which is the gradient of a potential ϕ , vanishing at infinity, and whose normal velocity component on the body surface cancels that due to the shear flow $u = Az$, $v = w = 0$ which results from the undisturbed vorticity distribution $(0, -A, 0)$.
- (2) the Biot-Savart velocity field, say V_1 of the vorticity change $\omega_1 = (\omega_x, \omega_y + A, \omega_z)$; the vorticity of the undisturbed flow has to be subtracted out or the Biot-Savart integral would not converge.

- (3) a further irrotational part, which is the gradient of a potential ϕ_2 vanishing at infinity, which cancels the normal velocity component on the body surface caused by the Biot-Savart velocity field.

The theory for uniformly sheared oncoming flow can be specialized further to the case of a two-dimensional primary flow in the (x, y) plane about a cylinder with generators in the z -direction.

The streamlines of the primary flow are written

$$y = y(x, y_0), \quad z = z_0 \quad (33)$$

and the time t is a function of x and y_0 alone, giving by (32), vorticity components in the form

$$\omega_x = Au \frac{\partial t}{\partial y_0}, \quad \omega_y = A(v \frac{\partial t}{\partial y_0} - \frac{\partial y}{\partial y_0}), \quad \omega_z = 0 \quad (34)$$

To simplify these equations, Lighthill used the streamfunction, Uy_0 , to express t as a function of x and y rather than x and y_0 . This transformation gives

$$\omega_x = AU \frac{\partial t}{\partial y}, \quad \omega_y = -AU \frac{\partial t}{\partial x}, \quad \omega_z = 0 \quad (35)$$

The exact secondary flow can be deduced from these equations by inspection, in three parts (1), (2) and (3) as given above. The potential ϕ_1 whose gradient cancels out the normal component of the shear flow $(Ay, 0, 0)$ on the body is

$$\phi_1 = Ay\phi \quad (36)$$

where

$$\phi = \text{the 'disturbance potential'}$$

The Biot-Savart field of the vorticity change

$$\omega_1 = (AU \frac{\partial t}{\partial y}, A - AU \frac{\partial t}{\partial x}, 0) \quad (37)$$

$$\text{is } u = 0, v = 0, w = A(x - Ut) \quad (38)$$

because the curl of (38) is (37) and its divergence vanishes. Finally no extra portion (3) is needed because the velocity field (38) is entirely in the z-direction.

Combining the secondary flows (36) and (38) with the primary flow and the oncoming shear flow, it can be seen that the velocity components in any plane $z = \text{constant}$ are given by

$$u = (U + Ay)(1 + \frac{\partial \phi}{\partial x}), v = (U + Ay)\frac{\partial \phi}{\partial y} \quad (39)$$

namely, the ordinary potential flow with the upstream velocity appropriate to that plane. Superimposed on these motions, however, is a flow parallel to the generators given by

$$w = A(x + \phi - Ut) \quad (40)$$

The distribution of w/Aa , calculated by Lighthill from Darwin's evaluation of t for a circular cylinder, for sheared flow about a circular cylinder of radius a , is shown in Fig. 5.5.

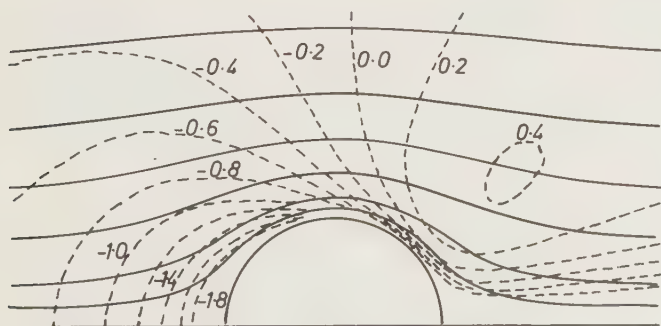


Fig. 5.5 Primary and secondary flow about a circular cylinder of radius a , with axis the z -axis, when the upstream velocity is $(U + Az, 0, 0)$

The greatest secondary flow velocities are in the direction of decreasing velocity physically, because this is the direction of decreasing stagnation-point pressure. Like t itself, the secondary flow velocity has theoretically a logarithmic infinity on the surface of the body; but viscous resistance must place a practical limit on the velocities which can be achieved.

To estimate the influence of viscosity on the secondary flow velocities near the wall, values of the secondary velocity computed from Lighthill's theory can be compared with values computed by Toomre (1959) for slightly viscous fluid.

Toomre's method applies to an infinitely long cylinder situated normally to a steady, incompressible, slightly viscous shear flow; the cylinder is also perpendicular to the vorticity, which is assumed uniform but small. The method is based on lateral pressure gradients, these being calculated from the primary flow alone. The method is only applicable to the plane of symmetry ahead of the cylinder.

Toomre calculated the secondary velocities ahead of a circular cylinder and ahead of a rectangular plate for four different values of the Reynolds number, $Re = \frac{Ua}{\nu}$, where a is the radius or halfwidth of the cylinder. His results are shown in Fig. 5.6.

Judging from Toomre's results, it seems that viscosity affects the flow pattern only in a very thin region. Even if the thickness of the viscous layer may be greater outside the plane of symmetry, it seems reasonable to assume that Lighthill's theory gives a good approximation of the flow, at least upstream of the points of separation.

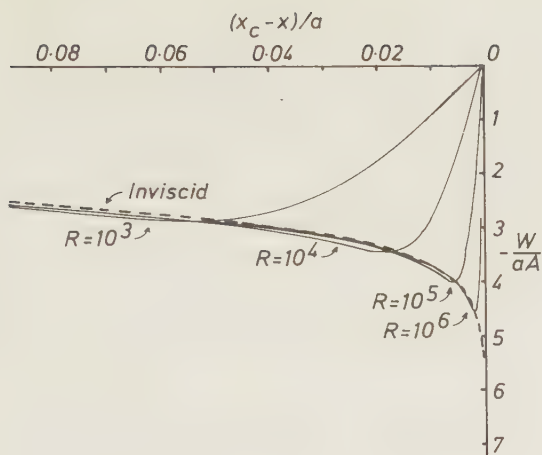


Fig. 5.6 Secondary flow velocities in the stagnation plane ahead of circular cylinder as calculated by Toomre

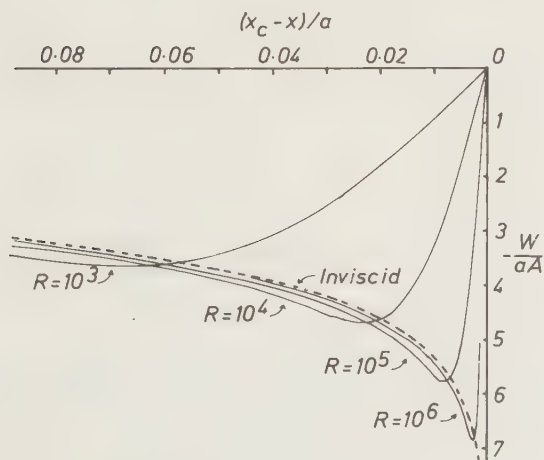


Fig. 5.7 Secondary flow velocities in the stagnation plane ahead of a rectangular cylinder as calculated by Toomre

5.4 A three-dimensional model

In a natural water course, the depth of flow is finite and the velocity profile has a non-constant gradient. Furthermore, the velocity gradient may be large compared to U/a , especially near the bed, where the approach velocity, U , can be small. Thus, for a natural situation, the assumptions underlying Lighthill's theory may be considerably violated near the bed and close to the cylinder. However, it is felt that an approach similar to that of Lighthill's may give, at least qualitatively, interesting information about the mechanics of the horse-shoe vortex system.

In line with what was done for the pipeline, the bed can be considered as a line of symmetry. Thus, the case to be dealt with is a flow about a cylinder where the velocity profile is symmetric with respect to $z = 0$. The situation is sketched in Fig. 5.8.

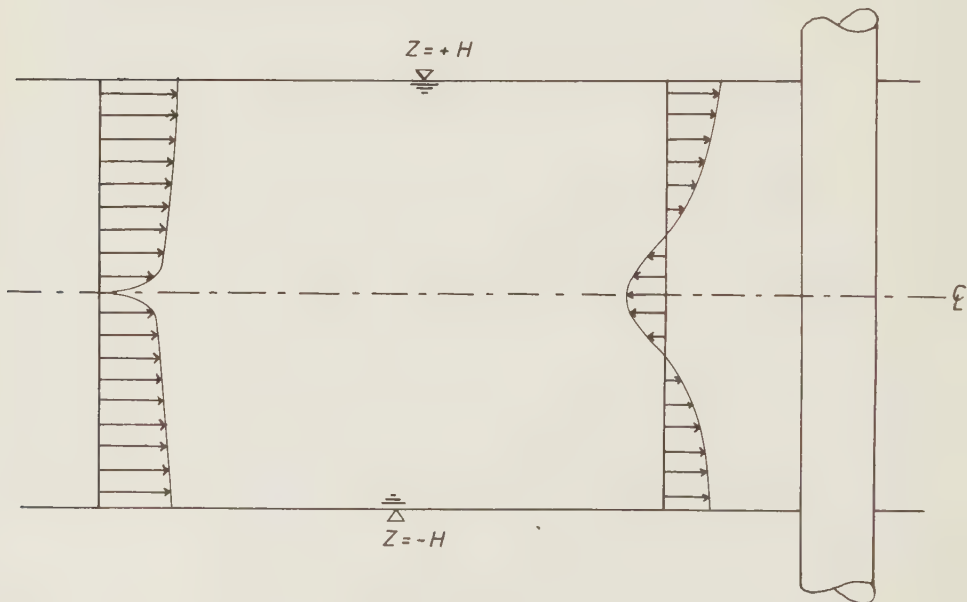


Fig. 5.8 Principle of symmetric flow

First it is assumed that the flow takes place in parallel planes, and that the local velocities in each plane can be calculated from the two-dimensional potential flow solution corresponding to the cross-section of the cylinder with the undisturbed velocity equal to the approach velocity at the level of the plane, $U(z)$.

Where

$$U(z) = \lim_{x \rightarrow \pm \infty} u(x, y, z) \quad (41)$$

If the cylinder is assumed to correspond to the disturbance potential ϕ , then the flow in any plane $z = z_0$ is given by

$$V(x, y, z_0) = U(z_0) \left[1 + \frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y} \right] \quad (42)$$

As in Lighthill's theory, the additional variable t is defined as

$$t(x, y_0, z_0) = \frac{x}{U(z_0)} + \int_{-\infty}^x \left(\frac{1}{u(x, y, z_0)} - \frac{1}{U(z_0)} \right) dx \quad (43)$$

Where the integration is to be performed along the streamline for which

$$\lim_{x \rightarrow -\infty} y = y_0, \quad \lim_{x \rightarrow -\infty} z = z_0 \quad (44)$$

The vorticity far upstream, $A(z_0)$, is given by

$$A(z_0) = \{dU(z)/dz\}_{z = z_0} \quad (45)$$

Thus, the vorticity field is given by

$$\left. \begin{aligned} \omega_x &= A(z_0) u(x, y, z_0) \frac{\partial t}{\partial y_0} \\ \omega_y &= A(z_0) \left[v(x, y, z_0) \frac{\partial t}{\partial y_0} - \frac{\partial y}{\partial y_0} \right] \\ \omega_z &= 0 \end{aligned} \right\} \quad (46)$$

Since $U(z_0)y_0$ is the stream function of the twodimensional flow in the plane $z = z_0$

$$\left. \begin{aligned} \frac{\partial y_0}{\partial x} &= -\frac{v(x, y, z_0)}{U(z_0)} \\ \frac{\partial y_0}{\partial y} &= \frac{u(x, y, z_0)}{U(z_0)} \end{aligned} \right\} \quad (47)$$

Substitution of (47) into (46) yields

$$\left. \begin{aligned} \omega_x &= A(z_0)U(z_0) \frac{\partial t}{\partial y} \\ \omega_y &= -A(z_0)U(z_0) \frac{\partial t}{\partial x} \\ \omega_z &= 0 \end{aligned} \right\} \quad (48)$$

As can be seen from (42) and (43)

$$\begin{aligned} t &= t(x, y_0, z_0) = \frac{x}{U(z_0)} + \int_{-\infty}^x \left(\frac{1}{U(z_0)(1 + \partial\phi/\partial x)} - \frac{1}{U(z_0)} \right) dx = \\ &= \frac{1}{U(z_0)} \left[x + \int_{-\infty}^x \left(\frac{1}{1 + \partial\phi/\partial x} - 1 \right) dx \right] = \frac{t_1(x, y_0)}{U(z_0)} \end{aligned} \quad (49)$$

Where

$t_1(x, y_0)$ corresponds to the drift function for unit approach velocity.

Thus, the vorticity change, ω_1 , can be written as

$$\omega_1 = A(z) \left[\frac{\partial t_1}{\partial y}, 1 - \frac{\partial t_1}{\partial x}, 0 \right] \quad (50)$$

According to the Biot-Savart law the associated secondary velocity field V_1 is given by

$$V_1 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-H}^H \frac{\omega_1 \times r}{4\pi|r|^3} dx dy dz \quad (51)$$

Where

r = the position vector relative to the element $dx dy dz$

Thus, for a general point (x_1, y_1, z_1) , the secondary velocity due to the vorticity change is given by

$$u_1(x_1, y_1, z_1) = \frac{1}{4\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(1 - \frac{\partial t_1}{\partial x}\right) \int_{-H}^H \frac{-A(z)(z - z_1)}{|r|^3} dx dy dz \quad (52)$$

$$v_1(x_1, y_1, z_1) = \frac{1}{4\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\partial t_1}{\partial y} \int_{-H}^H \frac{A(z)(z - z_1)}{|r|^3} dx dy dz \quad (53)$$

$$w_1(x_1, y_1, z_1) = \frac{1}{4\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[(x - x_1) \left(1 - \frac{\partial t_1}{\partial x}\right) - (y - y_1) \frac{\partial t_1}{\partial y} \right] \int_{-H}^H \frac{A(z)}{|r|^3} dx dy dz \quad (54)$$

It seems extremely difficult to find any general solution to the above expressions but they may well reveal qualitative properties of the flow field.

If the approach velocity is assumed to conform to the traditional boundary layer profile over a rough plate

$$\frac{u(z)}{u_*} = 2.5 \ln z/k_s + 8.5 \quad (55)$$

Where

u_* = the shear velocity $(\tau/\rho)^{1/2}$ and
 k_s = the equivalent boundary roughness

then

$$A(z) = dU(z)/dz = 2.5 u_{*}/z \quad (56)$$

Substitution of (56) into (52), (53), (54) gives

$$\frac{u_1}{u_*} = \frac{2.5}{4\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(1 - \frac{\partial t_1}{\partial x}\right) \int_{-H}^H \frac{-(z - z_1)}{z} \frac{dx dy dz}{|r|^3} \quad (57)$$

$$\frac{v_1}{u_*} = \frac{2.5}{4\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\partial t_1}{\partial y} \int_{-H}^H \frac{z - z_1}{z} \frac{dx dy dz}{|r|^3} \quad (58)$$

$$\frac{w_1}{u_*} = \frac{2.5}{4\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[(x - x_1) \left(1 - \frac{\partial t_1}{\partial x}\right) - (y - y_1) \frac{\partial t_1}{\partial y} \right] \int_{-H}^H \frac{1}{z} \frac{dx dy dz}{|r|^3} \quad (59)$$

Thus, the strengths of the flow fields V and V_1 are directly proportional to u_* .

Integration of (55) and rearranging yields

$$u_* = \bar{U} / (2.5 \ln^H / k_s + 6) \quad (60)$$

Where

$\bar{U}$ = the mean approach velocity

which shows that for a given configuration the flow pattern of $V + V_1$ is independent of the mean velocity.

Strictly, the water depth, H , affects as well the shear velocity, u_* , as the values of the integrals (57) to (59) but judging from

experimental evidence it should be more correct to evaluate the influence of H at constant u_* .

An exact evaluation is difficult to make, but from (56) and (52) to (54) it is obvious that the greater H is, the smaller becomes the influence of an increase of the depth.

By (55) the total vorticity of the approach flow in the region $z = 0$ to $z = H_1$ is equal to

$$u(H_1) = u_* \cdot 2.5 \ln H_1/k_s + 8.5 \quad (61)$$

An increase of water depth to $2H_1$ increases the total vorticity by

$$u(2H_1) - u(H) = u_* 2.5 \ln 2 \quad (62)$$

Thus, the relative increase of total vorticity induced by a twofold increase of water depth is

$$\Delta u = \ln 2 / (\ln H_1/k_s + 3.4) \quad (63)$$

If, for instance, the range of significant H_1 , is defined as the range within which Δu is greater than 10 % it is found that

$$0.10 = \ln 2 / (\ln H_1/k_s + 3.4) \quad (64)$$

or

$$H_1/k_s = 35$$

The value 10 % was somewhat arbitrarily chosen but it seems reasonable to assume that the range of significant water depths should be a function of the equivalent bed roughness k_s .

To estimate the influence of pier width, the equations (57) to (59) should be considered as written in length parameters normalized with respect to the pier width. Equation (42) may be similarly written as

$$\frac{V(x,y,z)}{u_*} = (2.5 \ln z + 8.5 + 2.5 \ln B/k_s) \left[\left(1 + \frac{\partial \phi}{\partial x}\right), \frac{\partial \phi}{\partial y} \right] \quad (65)$$

Thus, for a given set of dimensionless coordinates (x,y,z) , a specified pier shape, and for constant B/k_s , the relation between V and V_1 is constant, which means that the flow pattern is unaffected by a change of pier width if the ratio B/k_s is kept constant. However, normally the interest is centered on estimating the flow pattern about piers of different widths at a specified location and for this case k_s becomes a constant.

For constant k_s the strength of V increases with increasing B while V_1 is unaffected. Thus, the secondary flow becomes less pronounced as the pier width increases. However, as can be seen from (65) this effect is very small for normal ranges of B but it may certainly be important in model/prototype comparisons if the model has been run with sediment from the prototype location.

5.4.1 Flow around circular pier

To make an estimate of the validity of an approach along the above lines, numerical evaluations have been made for three different pier shapes, a circular cylinder, a semifinite rectangular pier, and a semifinite rectangular pier with a triangular nose whose included angle at the front was 90° .

For a circular cylinder, the well known disturbance potential is $\frac{a^2 x}{x^2 + y^2}$, where a is the radius of the cylinder.

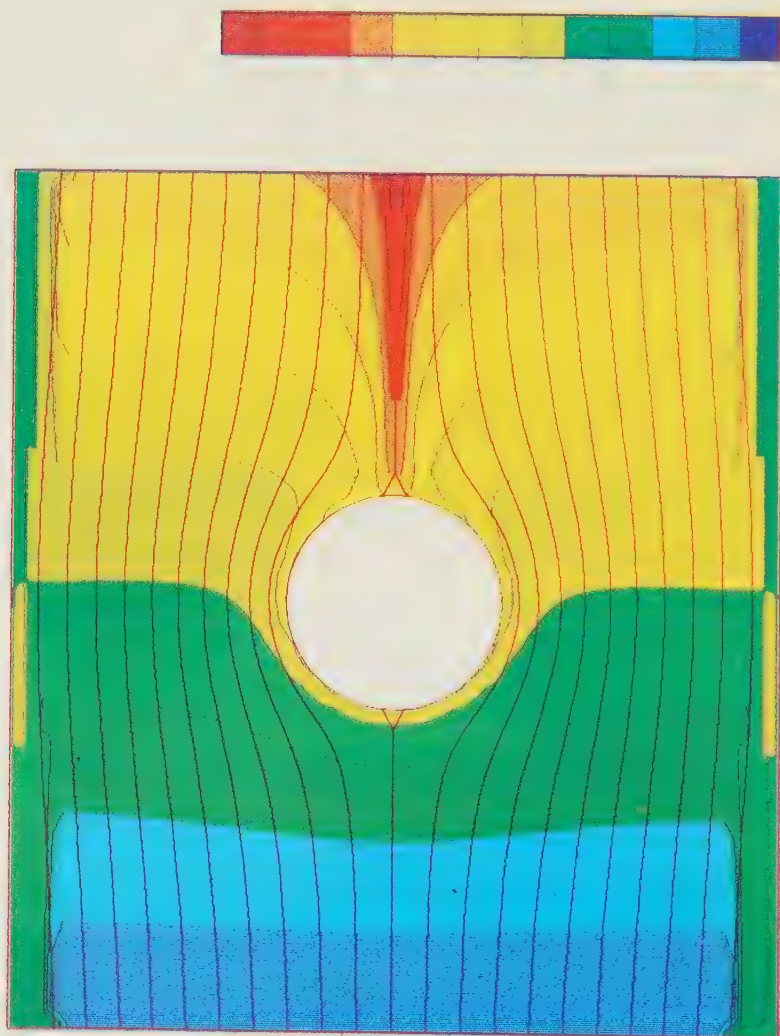
According to (42) and (43) the drift around a circular cylinder

is thus given by

$$t = \frac{1}{U} \left\{ x + \int_{-\infty}^x \left(\frac{1}{1 - \frac{a^2}{x^2 + y^2} + \frac{2a^2 y^2}{(x^2 + y^2)^2}} - 1 \right) dx \right\} \quad (66)$$

The calculated values of t are plotted in Fig. 5.9

Fig 5.9 Drift and streamlines around circular cylinder



5.4.2 Flow around rectangular pier

The potential function for flow around a semfinite rectangular shape is more complicated. To find this function, the principle of symmetry can again be employed. Thus, the flow over a step is equivalent to one half of the flow around the rectangle.

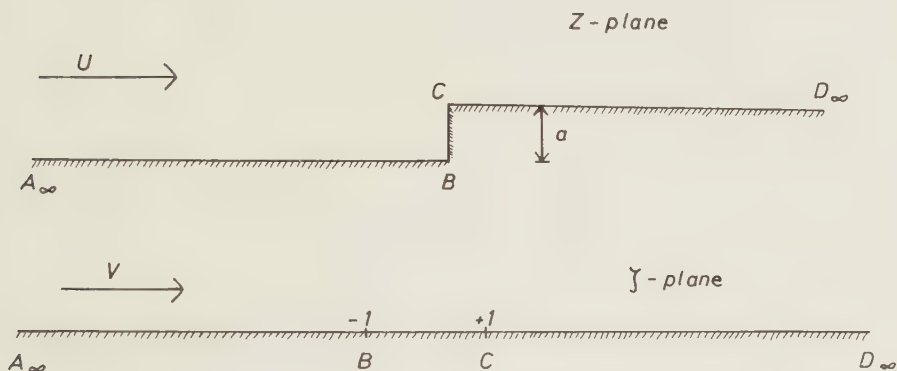


Fig. 5.10 Definition figure for the Schwartz-Christoffel transform

The bed is supposed to have an abrupt change of level at BC in a stream whose velocity at minus infinity is U . The height of the step is a . The points A_∞ and B_∞ are at infinite distances upstream and downstream respectively.

The bed $A_\infty BCD_\infty$ is a simple polygon and can therefore be transformed into the real axis of the ζ -plane, B and C corresponding to -1 and $+1$ respectively. By the Schwartz-Christoffel transformation [Milne - Thomson (1962)].

$$\frac{dz}{d\zeta} = K_R (\zeta - 1)^{-1/2} (\zeta + 1)^{1/2} \quad (67)$$

Where

$z = x + iy$, $\zeta = \xi + i\eta$, and K_R is a constant which may be complex

Integration yields

$$z = K_R \left[(\zeta^2 - 1)^{-\frac{1}{2}} - \cosh^{-1} \zeta \right] + L_R \quad (68)$$

Where

L_R = a constant of integration, which may be complex.

If, in the z -plane, B is taken as the point $(0, -a)$ and C as the point $(0, 0)$ then

$$K_R = \frac{a}{\pi}, \quad L_R = 0$$

so that

$$z = \frac{a}{\pi} (\zeta^2 - 1)^{1/2} - \cosh^{-1} \zeta \quad (69)$$

Next, the complex potential $\Omega = \phi + \psi$ is considered.

A uniform stream in the z -plane may be taken to imply a source at A_∞ and a sink of equal strength at D_∞ . In the ζ -plane there is then also a source at A_∞ and a sink at D_∞ , giving a uniform stream of velocity V in the ζ -plane. Thus the complex potential is

$$\Omega = V\zeta \quad (70)$$

and

$$\frac{d\Omega}{dz} = \frac{d\Omega}{d\zeta} \cdot \frac{d\zeta}{dz} = \frac{V\pi}{a} \left(\frac{\zeta + 1}{\zeta - 1} \right)^{1/2} \quad (71)$$

At minus infinity $\frac{d\Omega}{dz} = U \quad \zeta \rightarrow -\infty$

$$\text{hence } U = \frac{V\pi}{a}$$

$$\text{and } \Omega = \frac{Ua}{\pi} \zeta \quad (72)$$

Thus,

$$\frac{\partial \phi}{\partial x} = U \operatorname{Re} \left(\frac{\zeta + 1}{\zeta - 1} \right)^{1/2} . \quad (73)$$

and the drift around the rectangular pier is given by

$$t = \frac{1}{U} \left\{ x + \int_{-\infty}^x \left(\frac{1}{\operatorname{Re} \left(\frac{\zeta + 1}{\zeta - 1} \right)^{1/2}} - 1 \right) dx \right\} \quad (74)$$

The computed values of t for unit velocity and unit half-width of the pier are plotted in Fig. 5.11.

Fig 5.11 Drift and streamlines around rectangular cylinder



5.4.3 Flow around sharpnosed pier

The pointed nose pier can be treated similarly.

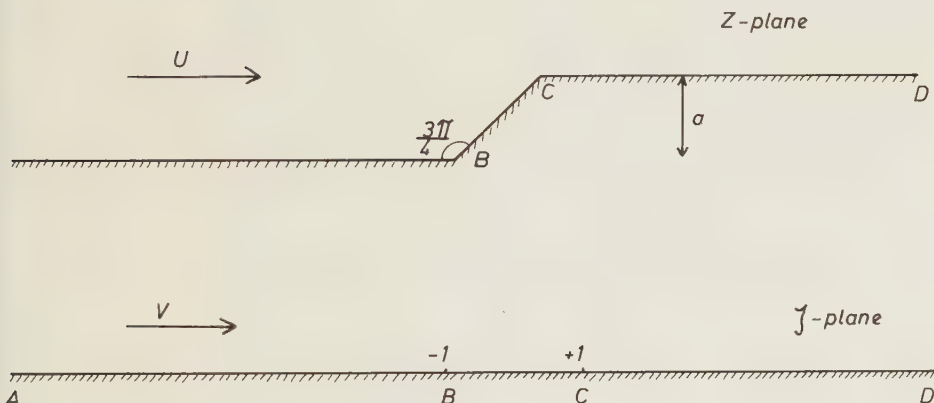


Fig. 5.12 Definition sketch for the Schwartz-Christoffel transform

For this case the Schwartz-Christoffel rule yields

$$\frac{dz}{d\zeta} = K_p (\zeta + 1)^{-\frac{1}{4}} (\zeta - 1)^{1/4} = K_p \left(\frac{\zeta - 1}{\zeta + 1} \right)^{1/4} \quad (75)$$

Where

K_p = a constant which may be complex

With

$$\zeta = \frac{1 + s^4}{1 - s^4} \quad (76)$$

the transform is changed into

$$\frac{dz}{ds} = K_p \frac{8s^4}{(1 - s^4)^2} \quad (77)$$

which can be evaluated to give

$$z = \frac{K_p}{2} \left[\frac{4s}{1-s^4} - 2 \arctan s + \ln \frac{1-s}{1+s} \right] + L_p \quad (78)$$

$$s = \left(\frac{\zeta - 1}{\zeta + 1} \right)^{1/4} \quad (79)$$

Where

L_p = a constant which may be complex.

The point B is transformed from the point $(-a, -a)$ of the z -plane to the point $(-1, 0)$ of the ζ -plane and C is transformed from $(0, 0)$ to $(1, 0)$.

The function $\left(\frac{\zeta - 1}{\zeta + 1} \right)^{1/4}$ is manyvalued and therefore the appropriate values have to be determined.

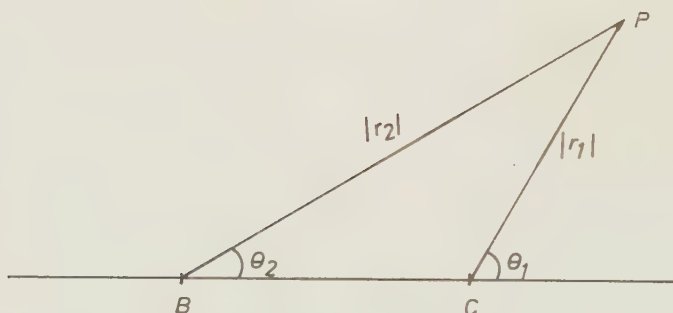


Fig. 5.13 Definition figure

Consider a general point P in the ζ -plane. The lengths of PC and PB are taken as r_1 and r_2 and therefore

$$\zeta - 1 = r_1 e^{i\theta_1}, \quad \zeta + 1 = r_2 e^{i\theta_2}$$

$$\left(\frac{\zeta - 1}{\zeta + 1} \right)^{1/4} = \left(\frac{r_1}{r_2} \right)^{1/4} e^{\frac{i}{4}(\theta_1 - \theta_2)}$$

If the point P is situated on the real axis between A_∞ and B then $\theta_1 = \theta_2 = \pi$ and thus,

$$\left(\frac{\zeta-1}{\zeta+1}\right)^{1/4} = \left(\frac{r_1}{r_2}\right)^{1/4} e^{\frac{1}{4}(\pi-\pi)} = \left(\frac{r_1}{r_2}\right)^{1/4} \quad (80)$$

Where

$$\left(\frac{r_1}{r_2}\right)^{1/4} \text{ signifies the positive root}$$

Accordingly:

$$\text{on BC } \left(\frac{\zeta-1}{\zeta+1}\right)^{1/4} = \left(\frac{r_1}{r_2}\right)^{1/4} \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) \quad (81)$$

and on CD_∞

$$\left(\frac{\zeta-1}{\zeta+1}\right)^{1/4} = \left(\frac{r_1}{r_2}\right)^{1/4} \quad (82)$$

Thus, $\zeta = -1$ corresponds to $s \rightarrow +\infty$

$\zeta = 1$ corresponds to $s = 0$

which yields $K_p = 2a/\pi$, $L_p = 0$

Now the complex potential is considered. A uniform stream in the z -plane corresponds to a source at A_∞ and a sink of equal strength at D_∞ . Thus, at the corresponding points in the ζ -plane there is also a sink at D_∞ and a source at A_∞ , giving a uniform velocity V in the ζ -plane. The complex potential is

$$\Omega = V\zeta \quad (83)$$

and

$$\frac{d\Omega}{dz} = \frac{V\pi}{2a} \left(\frac{\zeta+1}{\zeta-1}\right)^{1/4} \quad (84)$$

But for $x \rightarrow -\infty$ $\zeta \rightarrow -\infty$ $\frac{d\Omega}{dz} \rightarrow U$

and therefore $U = \frac{V\pi}{2a}$

Hence $V = \frac{2aU}{\pi}$

$$\text{and } \Omega = \frac{2aU}{\pi} \zeta \quad (85)$$

or in terms of the parameter s

$$\frac{d\Omega}{ds} = \frac{U}{s}$$

$$\frac{dz}{ds} = \frac{2a}{\pi} \frac{8s^4}{(1-s^4)^2} \quad (86)$$

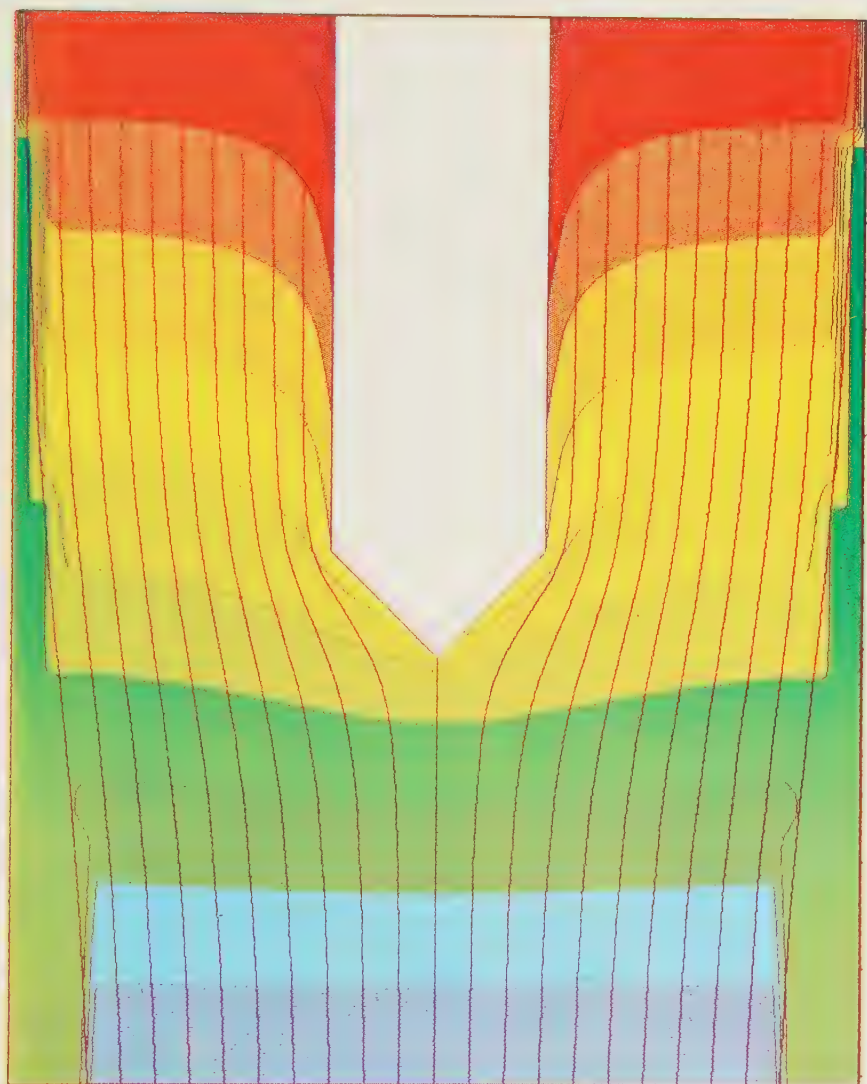
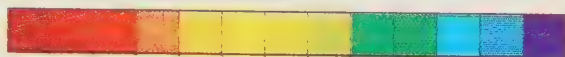
$$z = \frac{a}{\pi} \left[\frac{4s}{1-s^4} - 2 \cdot \arctan s + \ln \left(\frac{1-s}{1+s} \right) \right]$$

The drift function t becomes

$$t = \frac{1}{U} \left\{ x + \int_{-\infty}^x \left(\frac{1}{\operatorname{Re}(\frac{1}{s})} - 1 \right) dx \right\} \quad (87)$$

The calculated values of t are plotted in Fig. 5.14.

Fig 5.14 Drift and streamlines around sharp-nosed cylinder



The calculated drift functions (66), (74), (87) have been used to make numerical integrations of (57) to (59) with the following parameters

$$B = 2 \text{ m}, \bar{U} = 1 \text{ m/s}, H = 4 \text{ m}, k_s = 0.01 \text{ m}$$

The volume of integration was confined between a plane 3 m upstream of the pier nose and a plane 1 m downstream of the nose. The streamlines $\psi = \pm 3 U(z) \text{ m}^2/\text{s}$ were taken as lateral limits.

The step lengths were $dx = 0.10 \text{ m}$, $d\psi = 0.1 U(z)$ and vertically the fluid region was separated into ten layers the limits of which were taken at $z = 0$, $z = \pm 0.25 \text{ m}$, $z = \pm 0.50 \text{ m}$, $z = \pm 1 \text{ m}$, $z = \pm 2 \text{ m}$ and $z = \pm 4 \text{ m}$.

The calculated flows are sketched in Figs. 5.15 to 5.17. These show an encouraging qualitative correlation with observed flow phenomena. Of course the velocities at $z = 0$ are equal to zero in a real fluid but the calculated flow directions at $z = 0$ agree well with observed skin friction lines, see Figs. 6.4 and 6.7.

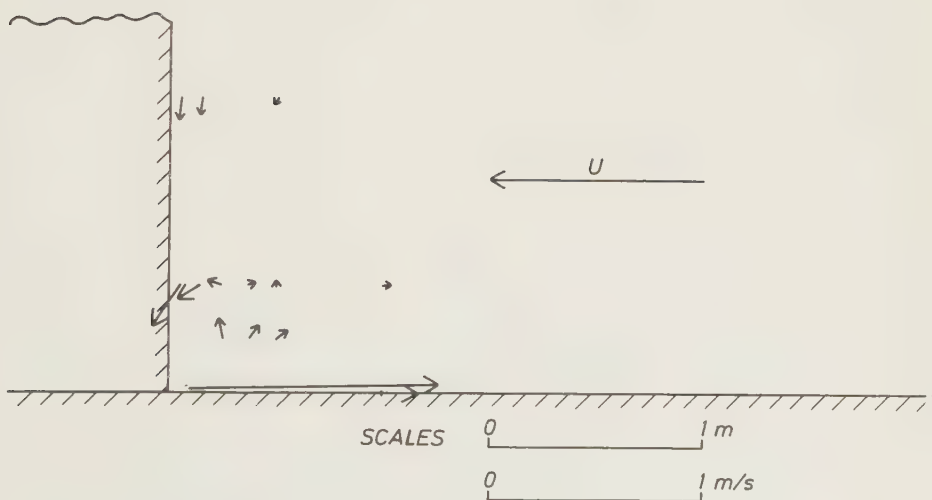
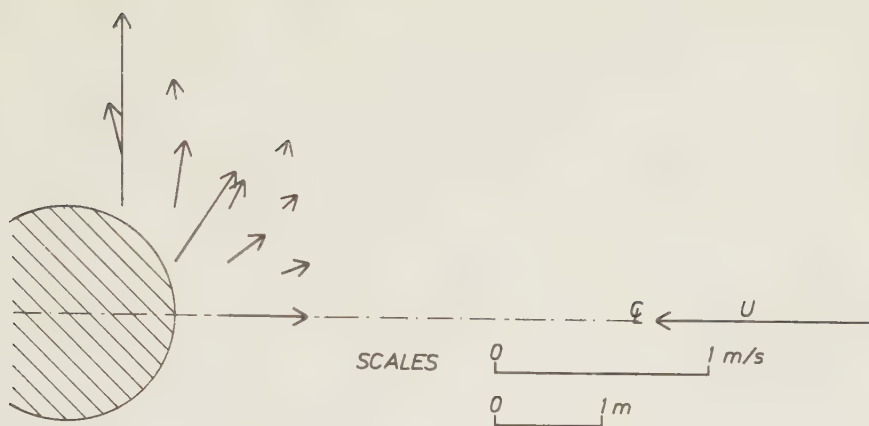


Fig. 5.15 Computed secondary flow near circular cylinder
 $U = 1$ m/s, $B = 2$ m, $H = 4$ m. Above: at $z = 0$,
 below: at $y = 0$

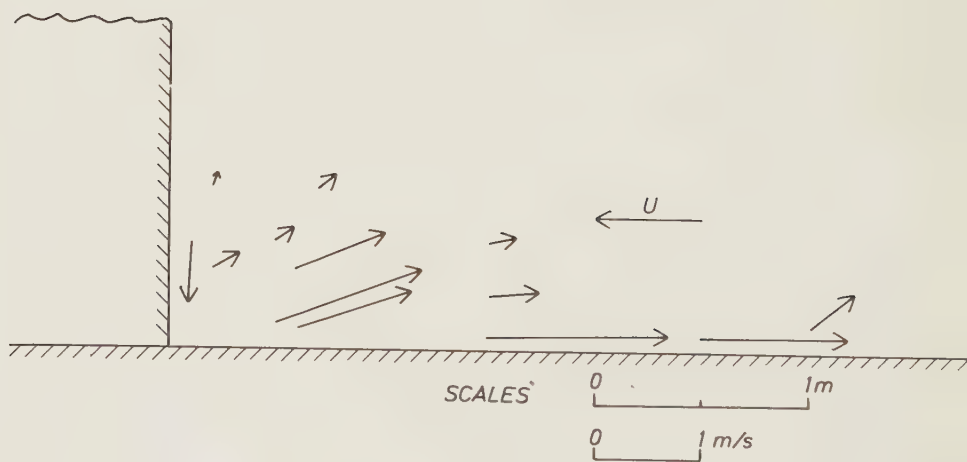
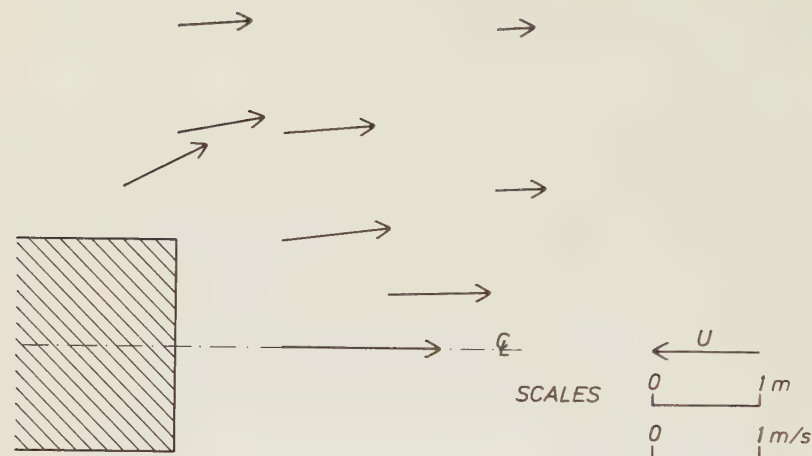


Fig. 5.16 Computed secondary flow near rectangular cylinder.
 $U = 1$ m/s, $B = 2$ m, $H = 4$ m. Above: at $z = 0$,
 below: at $y = 0$

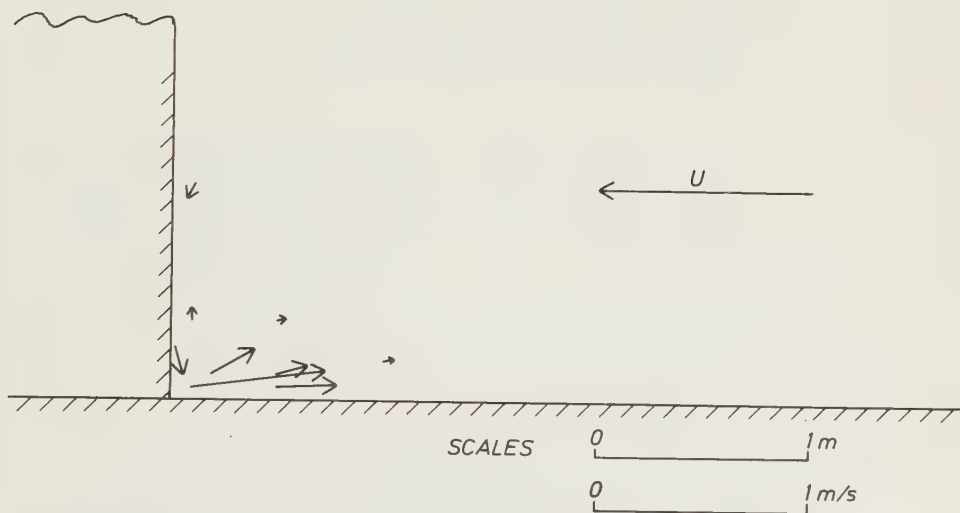
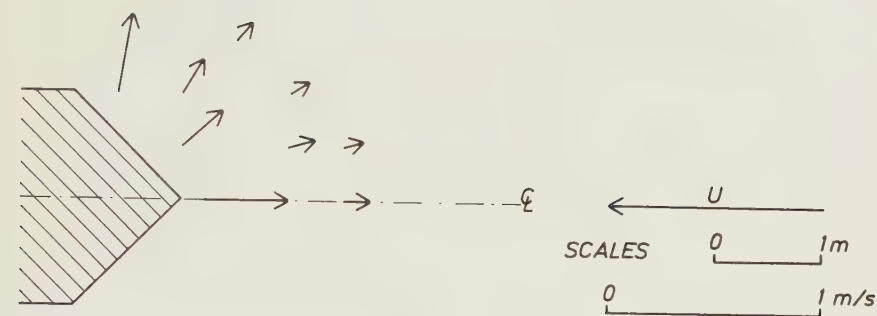


Fig. 5.19 Computed secondary flow near sharp-nosed cylinder.
 $U = 1$ m/s, $B = 2$ m, $H = 4$ m. Above: at $z = 0$,
 below: at $y = 0$

5.5 Turbulent effects

Among other important processes in flows about cylinders are vortex shedding and the wake flow downstream of the cylinder.

Vortex shedding is mostly studied for the two-dimensional flow around cylinders. The purely analytic studies are confined to a region of very small Reynolds numbers. For higher values of the Reynolds number, only empirical knowledge is available.

For circular cylinders the shedding has been found to depend on the state of the boundary layer on the upstream side of the pier. Three different cases have been identified.

- (1) the boundary layer is laminar. The point of separation is moved upstream as velocity increases
- (2) a separation bubble is formed. This means that the boundary layer separates but is reattached again just before the final separation. The point of separation is considerably farther downstream than for case (1)
- (3) the boundary layer is turbulent and has an increased ability to withstand adverse pressure gradients, compared to a laminar boundary layer. In this case the point of separation is farther downstream than for laminar boundary layers

An approximate relationship between Reynolds number and angle of separation is shown in Fig. 5.19

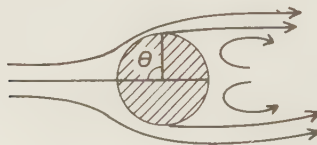


Fig. 5.18 Separation at circular cylinder

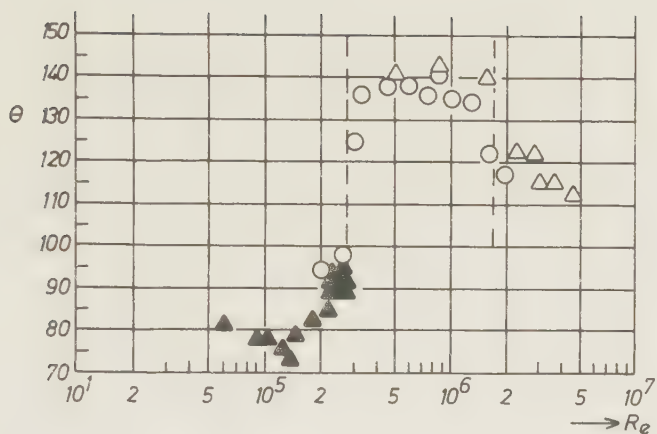


Fig. 5.19 Angle of separation at circular cylinder as function of Re . Δ Height/width 3.33 ND channel; $\blacktriangle$ Height/width 3.33 HD channel, $\circ$ Height/sidth 6.66 HD channel. After Achenbach (1968)

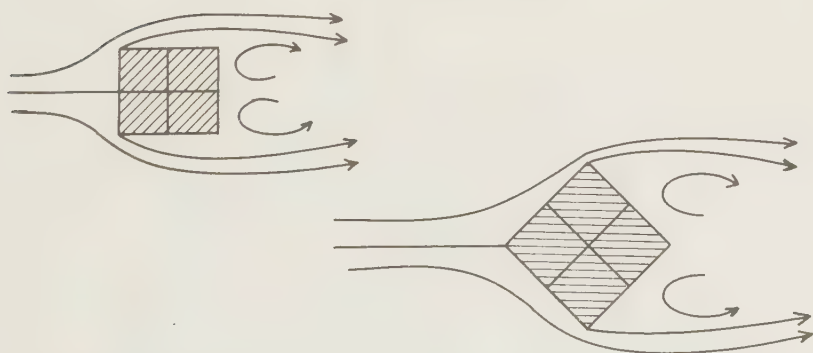


Fig. 5.20 Schematic illustration of separation at sharpedged cylinders

For sharp-edged profiles, separation is induced at the edges, Fig. 5.20. The vortex shedding seems to be periodic over a large range of Reynolds number. For circular cylinders four characteristic regions have been found.

- (1) the subcritical region. The vortex shedding frequency corresponds to the Strouhal number $Sh = \frac{f_s B}{U} \approx 0.2$, where f_s is the shedding frequency, B the diameter of the cylinder and U the undisturbed approach velocity. This region is given by $Re < 1.5 \cdot 10^5$
- (2) the critical region. In this region the Strouhal number increases with Re from 0.2 at $Re = 1.5 \cdot 10^5$ to about 0.4 at $Re = 2.5 \cdot 10^5$
- (3) the overcritical region. The vortex shedding is no longer periodic. This region is characterized by $2.5 \cdot 10^5 < Re < 10^6$
- (4) the transcritical region. The vortex shedding again becomes periodic and is characterized by a Strouhal number of approximately 0.2

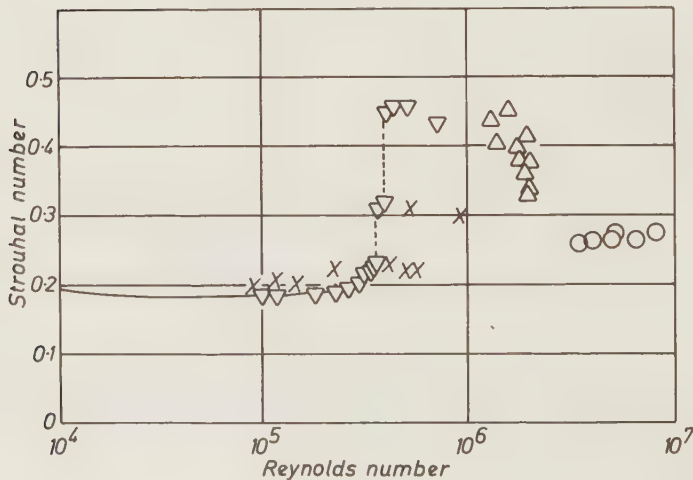


Fig. 5.21 Empirical relation between shedding frequency and Reynolds number for cylinders in two-dimensional flow. ∇ Bearman (1969), x Relf and Simmons (1924), Δ Delany and Sorensen (1953), o Roshko (1961)

To what extent these results apply to three-dimensional flows with a spanwise velocity gradient is not known. In this case; the approach velocity varies and a constant value of Sh would imply different shedding frequencies at different levels. An investigation by Shaw and Starr (1972) indicates that the regular shedding behaviour of two-dimensional flows does not exist for three-dimensional velocity gradient flows.

The flow in the region of separation has been studied by Son and Hanratty (1969) for two-dimensional flow around a cylinder at Reynolds numbers from $5 \cdot 10^3$ to 10^5 .

In the region before separation, they found the average velocity gradient to have a fluctuating component superimposed upon it. In the rearward portions of the cylinder this periodic fluctuation was obscured by random oscillations.

It was found that boundary layer theory gives satisfactory agreement with observations and that the shear stress distribution in the neighbourhood of the front stagnation point agrees with predictions based on an external flow given by classical irrotational analysis.

Over the front portion of the cylinder Son and Hanratty found the oscillations to be sinusoidal and of constant frequency. The amplitude of the oscillations was found to be quite large near the separation point. At, or near, the separation point irregular fluctuations were imposed upon the sinusoidal ones. Farther from separation the fluctuations became more irregular and as well the amplitude as the frequency increased. The frequencies, f , of the regular fluctuations were plotted as $Sh = f^3/U$ and compared with measurements of flow fluctuations in the wake by Roshko and Gerrard, Fig. 5.22. The results show good agreement.

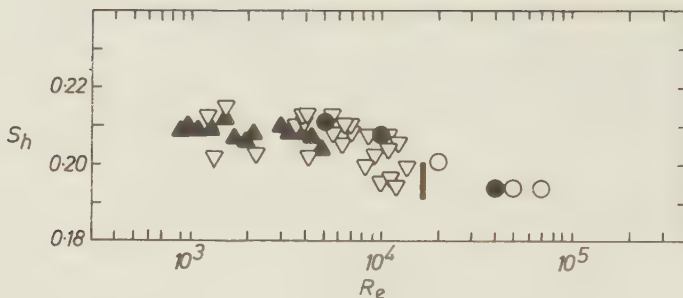


Fig. 5.22 Fluctuation frequencies in the wake behind a two-dimensional cylinder.

● Son and Hanratty ▲ Roshko ■ Gerrard

Son and Hanratty also tried to figure out the flow pattern. A possible flow picture is indicated in Fig. 5.23. The region B signifies a small relatively stationary secondary vortex after the separation point. The gross wake structure is pictured as oscillating between that pictured in Fig. 5.23 a and that pictured in Fig. 5.23 b.

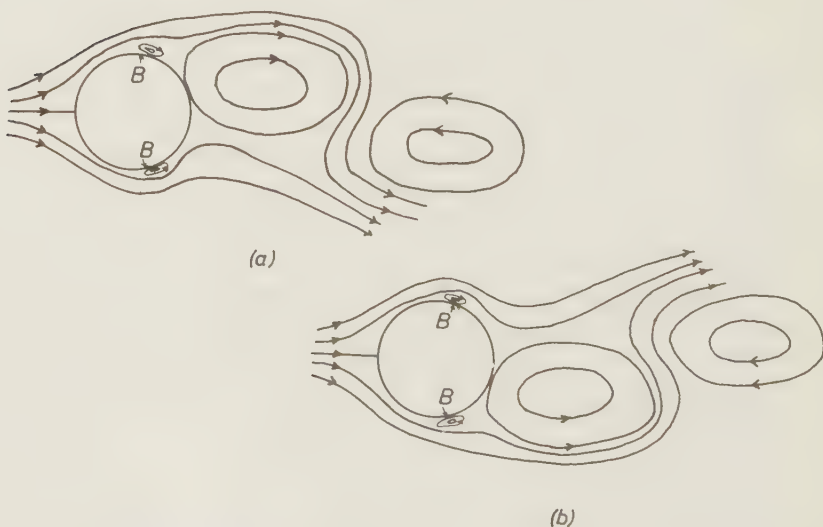


Fig. 5.23 Possible flow pattern behind two-dimensional circular cylinder according to Son and Hanratty (1969)

According to this model the flow immediately after the secondary vortex is in the positive direction but, farther downstream, it alternates its direction with time. In the rear of the cylinder, highly complicated three-dimensional flow fluctuations are superimposed on this gross structure.

The effect of turbulence of the approach flow is not known. According to Shen et al (1966) it can be presumed to be small. However, Roberson, Lin, Rutherford and Stine (1972) found that for some shapes turbulence may influence the two-dimensional flow pattern. They examined the flow pattern about rectangular rods and found that the streamlines may move in closer to the body with increased turbulence.



Fig. 5.24 Effect of turbulence intensity on flow pattern about rectangular rod. (a) turbulence intensity 0.5 %, (b) turbulence intensity 8 %. After Roberson et al (1972)

Their conclusions were

- (1) Changes of the free stream turbulence intensity cause changes in Reynolds stresses and flow pattern about a body.
- (2) Bodies which have shapes such that reattachment of flow is not a factor, experience an increase in drag coefficient, C_D , with increased turbulence level.
- (3) Bodies for which reattachment or near reattachment of flow occurs with increased turbulence may experience

either a decrease or an increase in C_D with increased turbulence level, depending on the shape of the body,

- (4) Bodies most sensitive to the effect mentioned in conclusion (3) are bodies whose length to width ratio is close to unity.
- (5) Pressure distribution on the side of bodies of the types noted in conclusions (3) and (4) may change as much as 100 % with a change in turbulence intensity of about 10 %.
- (6) Turbulence in the free stream also accentuates unsteady dynamic forces on certain bodies.

6 EXPERIMENTAL INVESTIGATION

6.1 Flow pattern and velocity distribution

The first part of this local scour study was an investigation of the flow pattern and velocity distribution of flows about cylinders and pipes in a rigid boundary model test series. The results have been presented in a previous publication, Hjorth (1972). The techniques used were hot film anemometry to determine mean velocities and turbulent velocity fluctuations, and a plaster of Paris model technique to determine the mean streamlines of the flow in the wall region. The results are summarized below.

6.1.1 Cylindrical piers

Two cylinders, with diameters, B , 3.8 cm and 8.4 cm, were used for the investigation. The velocity measurements were mostly confined to a plane through the axis of the pier and normal to the approach flow. As could be expected, the horizontal velocity gradients close to the cylinders could be very steep. More important is the effect of the cylinder on the vertical velocity gradient close to the pier. These profiles were found to be almost rectangular instead of having the generally assumed form of the equilibrium boundary layer profile. This indicates that the region of vorticity has been concentrated by convection to a thinner layer of higher vorticity. Together with the increase of mean velocity over the profile, this effect may lead to a sizeable amplification of the boundary shear. The effect was negligible at distances greater than $2 B$ from the cylinder.

The horizontal velocity gradient in the upper layers of the flow was found to be in fair agreement, outside the viscous layer close to the pier, with that predicted by classical two-dimensional potential flow theory.

Measurements of the vertical velocity profile in the plane of symmetry upstream of the cylinders showed that, as the flow approaches the cylinder, its presence is first affecting the lower layers of the flow, at a distance of about $4B$. As the flow approaches the cylinder the level of noticeable influence increases continuously, Fig. 6.1.

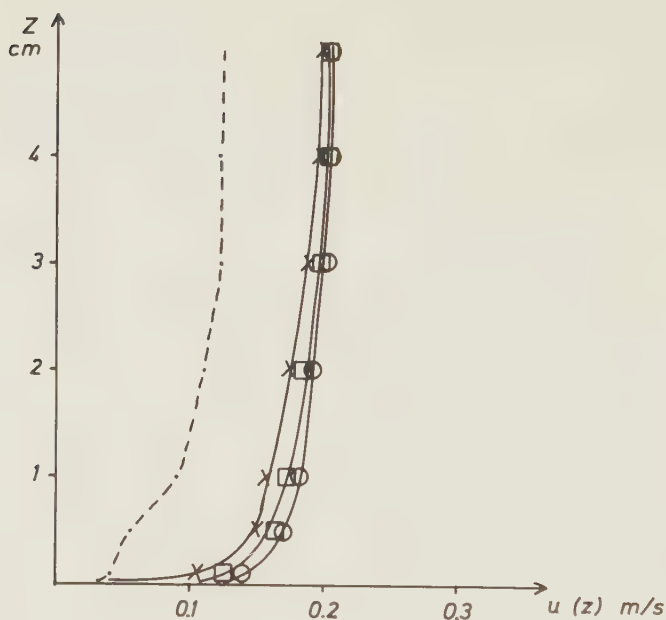


Fig. 6.1 Velocity profiles in the stagnation plane upstream of a 8.4 cm circular cylinder • 2 cm upstream, x 7 cm upstream, □ 17 cm upstream, ○ 32 cm upstream

The turbulence intensity is increased in a region close to the pier and is uniformly distributed over the vertical. As the distance from the cylinder increases the turbulence intensity decreases and its vertical distribution tends towards the

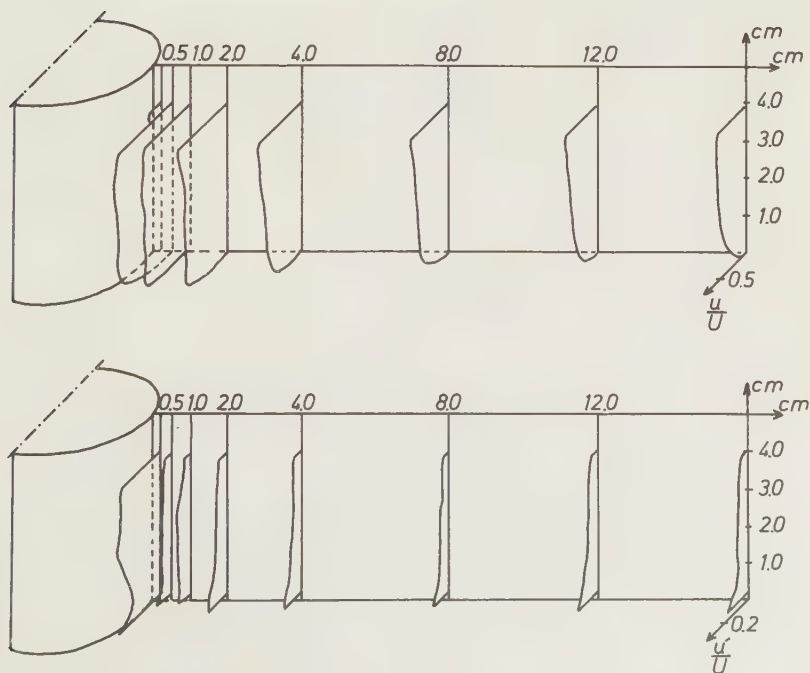
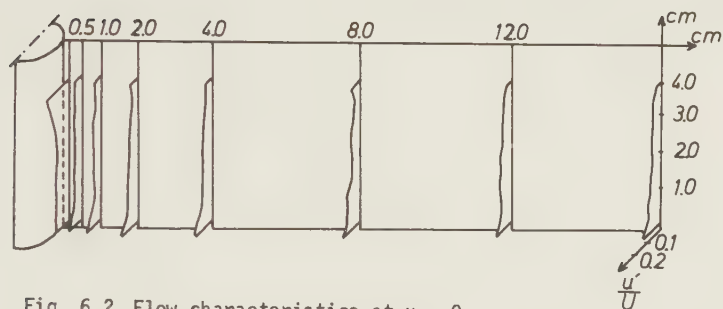
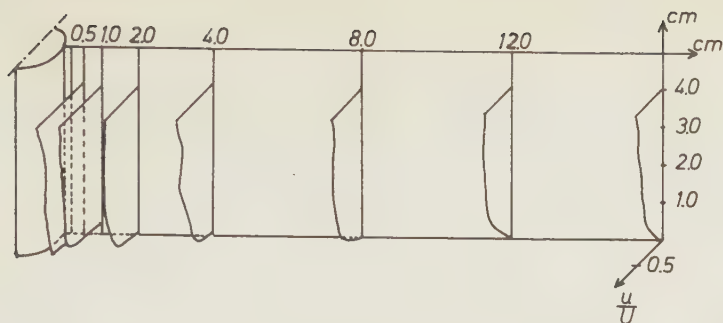


Fig. 6.3 Flow characteristics at $x = 0$
 $U = 21.7 \text{ cm/s}$, $H = 9.2 \text{ cm}$, $B = 8.4 \text{ cm}$

distribution of an equilibrium boundary layer profile, that is, the intensity has its maximum at the bed.

Some results are shown in Figs. 6.2 and 6.3.

The plaster models showed that the streamlines of the wall layer are deflected around the cylinder with the closest streamline passing about one cylinder radius away from the cylinder. Fig. 6.4.

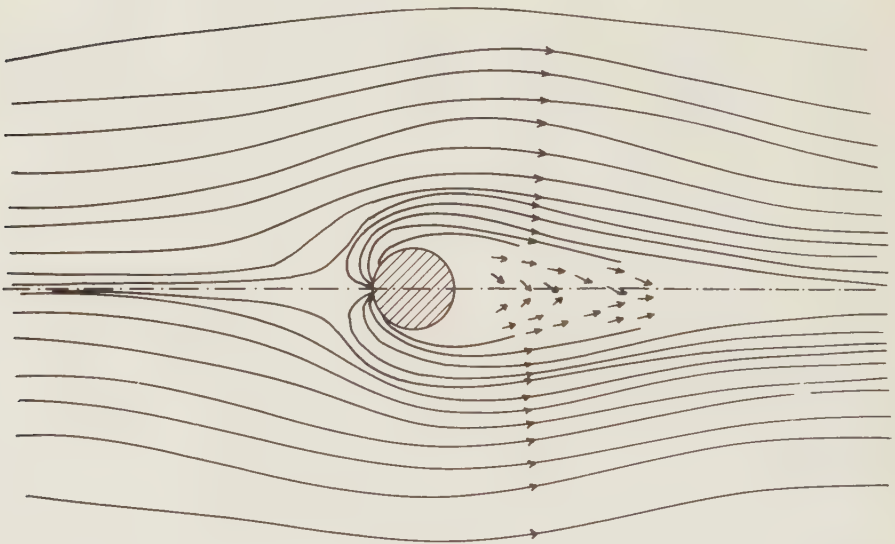


Fig. 6.4 Streamlines in the wall region near a circular cylinder

Thus there is a region around the cylinder into which water is fed from higher layers. Downstream of the cylinder, the width of this zone diminishes and about 5 or 6 B downstream of the cylinder the streamlines have reattained their approximate lateral positions from the undisturbed flow. The streamlines on the upstream front of the cylinder are similar to the ones on the downstream part. The flow is from the stagnation line towards

the lines of separation, though the flow on the downstream part of the cylinder has a more marked downward component, Fig. 6.5.

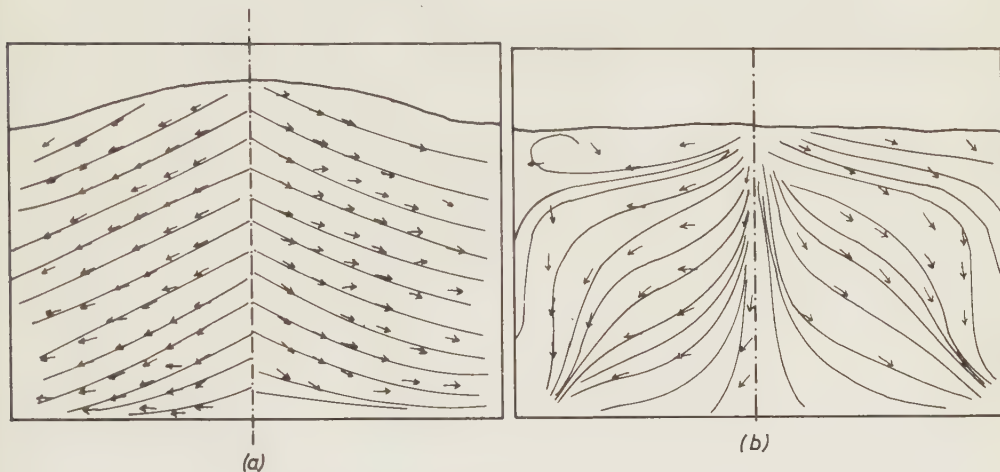


Fig. 6.5 Streaming in the wall region on the surface of a circular cylinder
(a) upstream half
(b) downstream half

A picture of the gross flow pattern is sketched in Fig. 6.6.

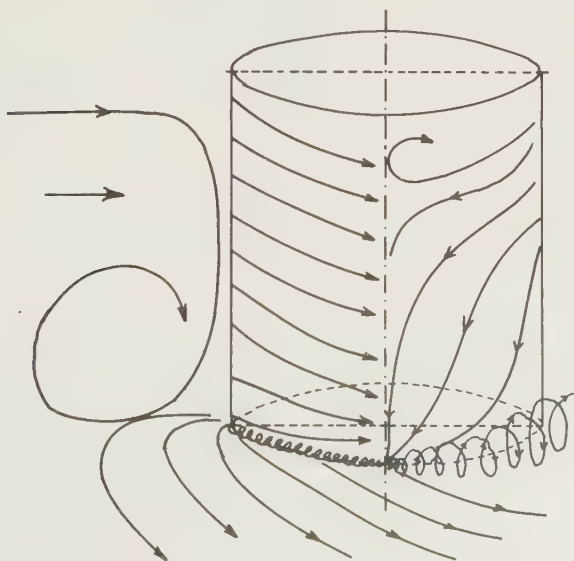


fig. 6.6 Schematic picture of flow about circular cylinder

6.1.2 Rectangular cylinders

Two cylinders, with cross-sections $3.8 \times 3.8 \text{ cm}^2$ and $8.4 \times 8.4 \text{ cm}^2$ were used.

The typical pattern of the limiting streamlines is shown in Fig. 6.7.

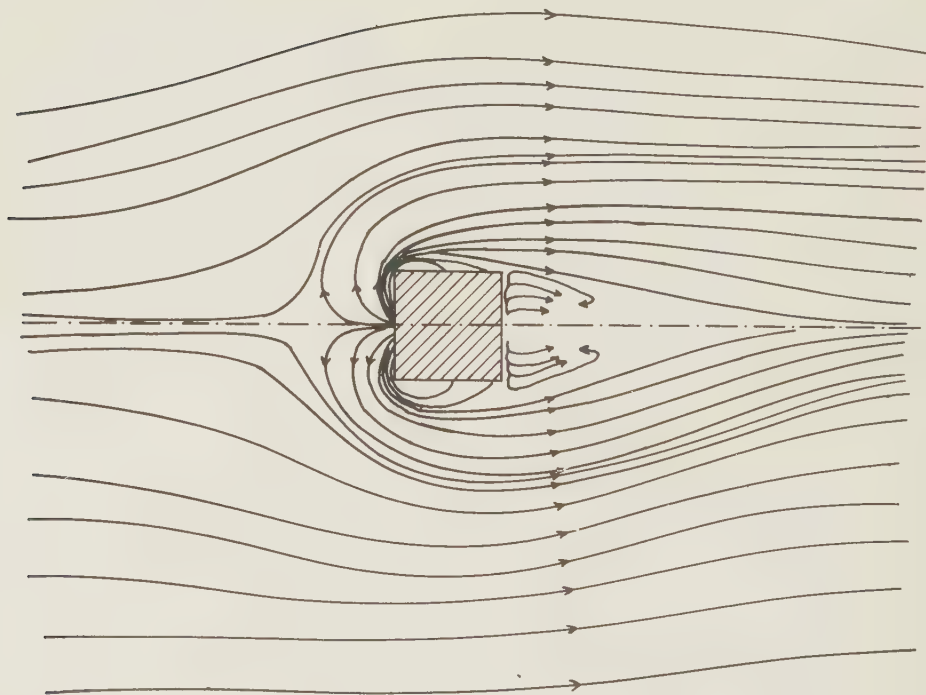


Fig. 6.7 Limiting streamlines around rectangular cylinder

For this case, the region of secondary flow is about 50 % wider than for a circular cylinder of the same width. Outside this region the flow has the same main features as for circular piers.

The flow patterns on the upstream and downstream cylinder fronts are similar but as for circular cylinders the downward component is somewhat more marked on the downstream side. The pattern on the sides of the cylinders indicate a complex vortex motion. Fig. 6.8. A schematic picture of the vortex motion near the cylinder is sketched in Fig. 6.9

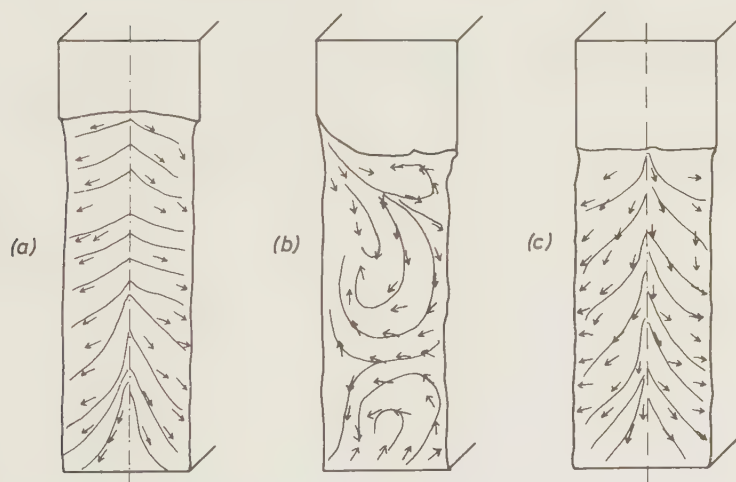


Fig. 6.8 Flow pattern on the surface of a rectangular cylinder
 (a) front
 (b) side
 (c) back

The horizontal velocity profiles are similar to the horizontal profiles for circular cylinders except near the cylinder. Here the rectangular cylinder gives a zone of secondary flow between the cylinder and the primary flow. This is because the rectangular shape forces the flow to separate already at the upstream corner.

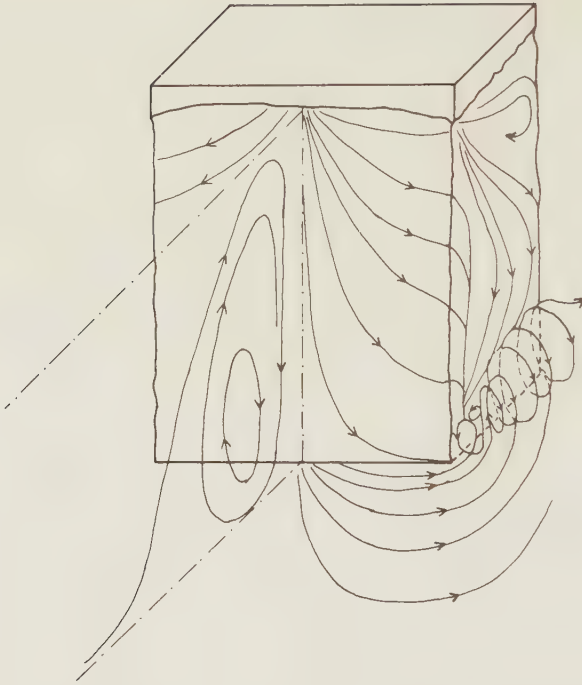


Fig. 6.9 Schematic picture of flow about rectangular cylinder

The vertical velocity profiles near the cylinder are quite strange. Within the wake they have a negative gradient of considerable magnitude. Probably this phenomenon is caused by suction from the vortex motion near the cylinder. Farther towards the primary flow, the vertical velocity becomes more rectangular and near the separation surface it is similar to the profile found for circular cylinders. As for these the normal boundary layer profile is approached as the distance from the cylinder increases. Figs. 6.10 and 6.11. show some typical results.

The flow pattern did not change with depth of water nor was it influenced by velocity changes. However, the strength of the flow was found to be proportional to the approach velocity.

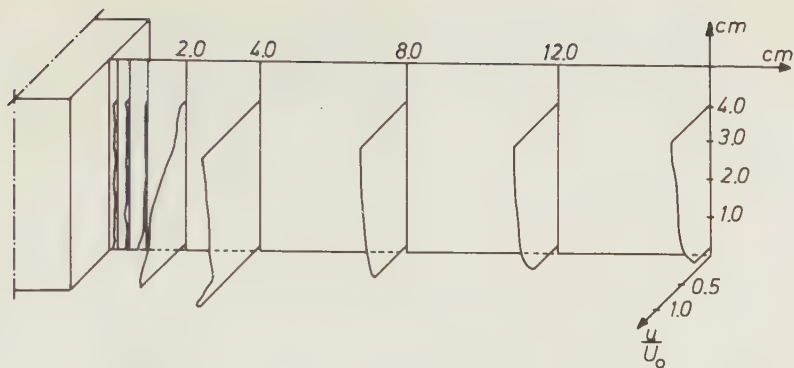


Fig. 6.10 Flow characteristics at $x = 0$
 $U = 21.7$ cm/s, $H = 5.0$ cm, $B = 8.4$ cm

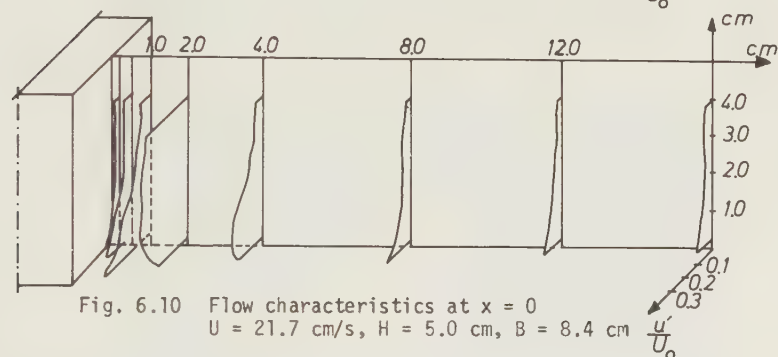
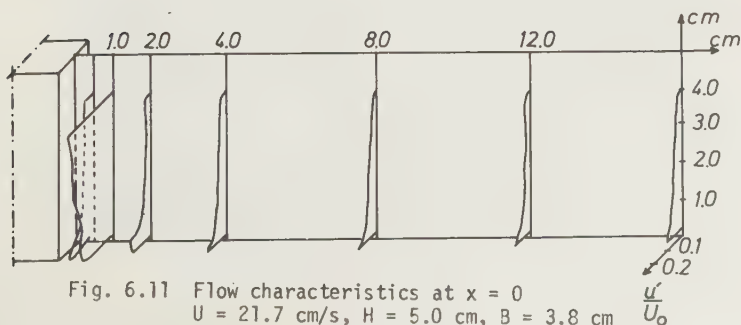
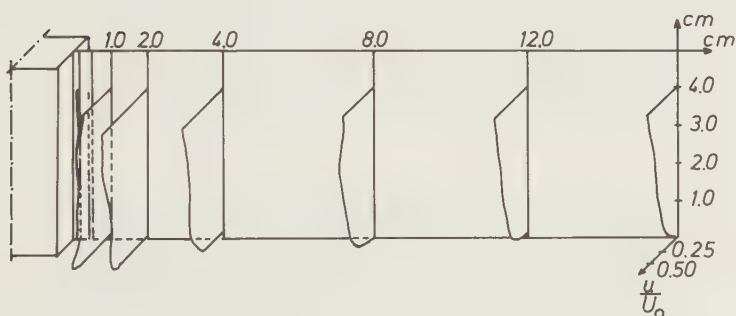


Fig. 6.11 Flow characteristics at $x = 0$
 $U = 21.7$ cm/s, $H = 5.0$ cm, $B = 3.8$ cm



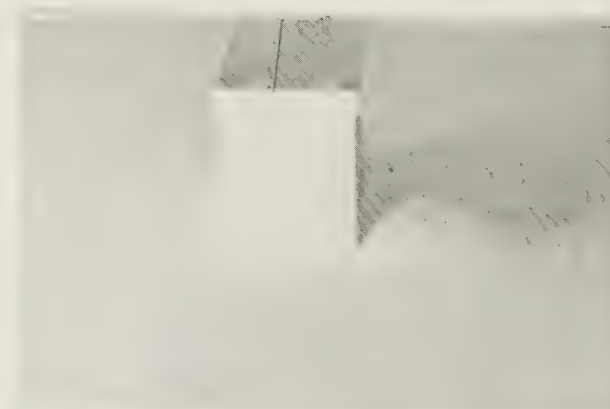


Fig. 6.12

For some points spectral- and auto-correlation functions were computed. These showed that there is a fluctuating component which is caused by the vortex shedding but the flow is inherently turbulent so the corresponding peak of the energy spectrum is hardly of practical importance.

6.1.3 Devices to weaken the horse-shoe vortex

Some plaster models were run to investigate the effects on the horse-shoe vortex system invoked by some simple arrangements at the foot of the cylinder. The main idea was to add a small structure at the pier footing to guide the flow more smoothly around the pier. The tests showed, that it is indeed possible to weaken the vortex flow on the bed. However, care must be taken that the additional structure does not itself create any secondary vortices. Some examples are shown in Fig. 6.12.

6.1.4 Effect of turbulence level of the approach flow

A few very simple tests were made to investigate if the approach flow turbulence level had any influence on the flow pattern near the cylinder.

Pairs of identical plaster models were run under similar conditions except that upstream of the first model roughness elements were introduced to increase the turbulence level while the boundary upstream of the second model was smooth. Comparison of flow patterns showed no significant differences between the models exposed to flows with high turbulence levels and models exposed to flows with lower turbulence levels.

Thus these experiments did not contradict the assumption of Shen et al (1966) that the influence of approach flow turbulence is small. However, the findings of Roberson et al (1972) are not necessarily contradicted either since they compared turbulence levels of order 1 % to intensities of order 10 %. Their lowest turbulence level is extremely low and their high level corre-

sponds to the low level of the abovementioned experiments. Thus, for natural flows, the effect of turbulence level may be assumed to be small.

6.1.5 Flow over pipelines

An example of the observed flow patterns is shown in Fig. 6.13.

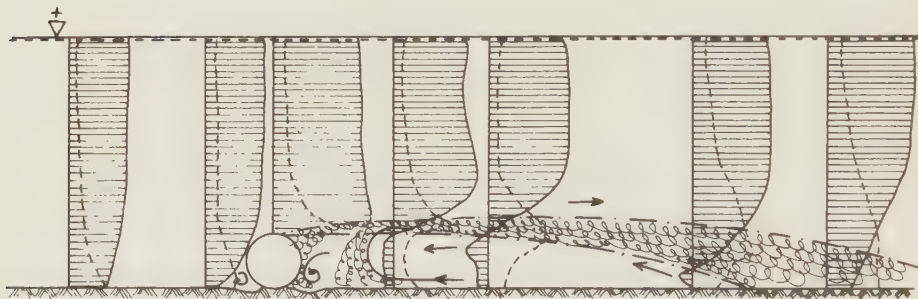


Fig. 6.13 Observed flow over a pipeline at a right angles to the oncoming flow

A characteristic feature of the flow is the development of three vortices, one under the upstream separation surface, and two, one primary and one secondary, under the downstream surface of separation. The distance from the pipe of the upstream line of separation is approximately equal to the height of the crest over the general bed surface. The distance from the pipe to the downstream reattachment line is approximately six times the height of the pipe crest. Thus the upstream vortex is supposed to be near circular while the downstream primary vortex has a length to width ratio of about five. However, judging from the plaster model the secondary vortex seems to have a form corresponding to the upstream vortex.

The flow under the surfaces of separation could not be measured with the hot-film anemometer and no conclusions could be drawn about the strength of the vortex flows. Some results from the

hot-film measurements outside the vortex regions are shown in Figs. 6.14 to 6.16.

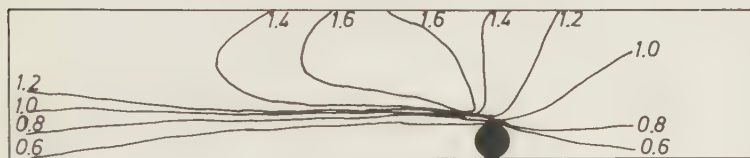


Fig. 6.14 Velocity distribution about pipe resting on the bed
The lines indicate the value of u/U_∞ . Flow from right to left.
 $U_\infty = 21.7$ cm/s
 $B_\infty = 2$ cm
 $H = 14$ cm



Fig. 6.15 Velocity distribution about pipe halfway embedded
 $U_\infty = 21.7$ cm/s
 $H_\infty = 14$ cm
 $B = 4$ cm

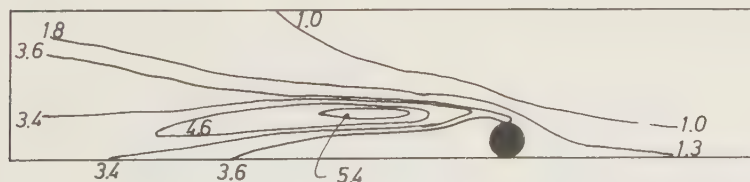


Fig. 6.16 Distribution of turbulence about pipe resting on the bed. The lines indicate the ratio of local turbulence to mean turbulence of the approach flow. Flow from right to left.
 $U_\infty = 21.7$ cm/s
 $H_\infty = 14$ cm
 $B = 2$ cm

6.2 Boundary shear stresses around cylinders and pipelines

The second phase of the experimental program was an investigation of boundary shear distribution around the types of obstacles under consideration. The experiments, which have been reported elsewhere, Hjorth (1974), were performed in a rigid boundary flume. The results were in certain respects contrary to expectations and are summarized below.

6.2.1 Circular cylinders

Two cylinders were used, with diameters $B = 5$ cm and $B = 7.5$ cm. Two different velocities, 15 cm/s and 30 cm/s, and two depths of approach flow, 10 cm and 20 cm, were investigated. The position of the measuring points is illustrated in Fig. 6.17.

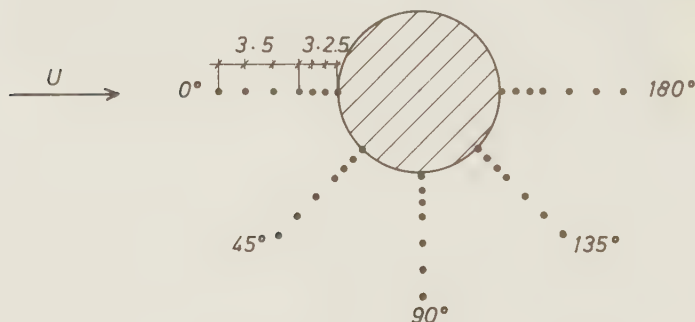
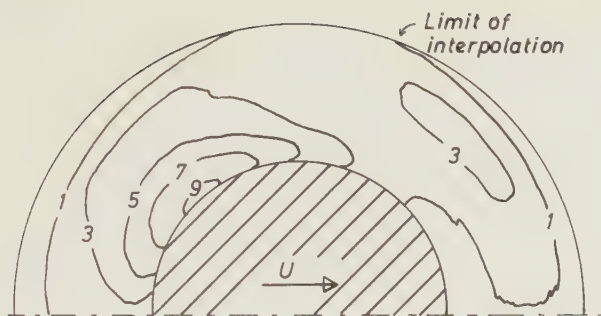


Fig. 6.17 Location of points of observation around circular cylinder

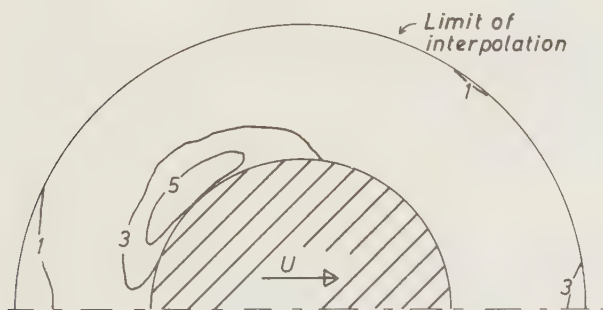
The observed values of boundary shear stress were interpolated to give the shear stress distributions shown in Figs. 6.18 to 6.20 where the lines represent equal amplification of the boundary shear of the approach flow.



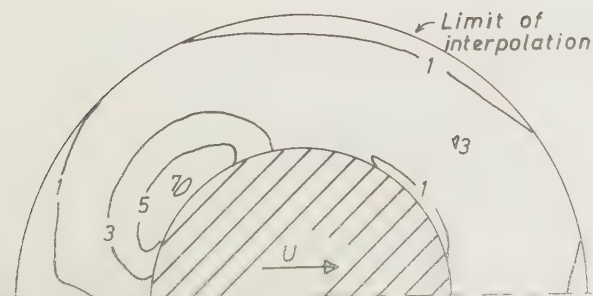
(a)



(b)



(c)



(d)

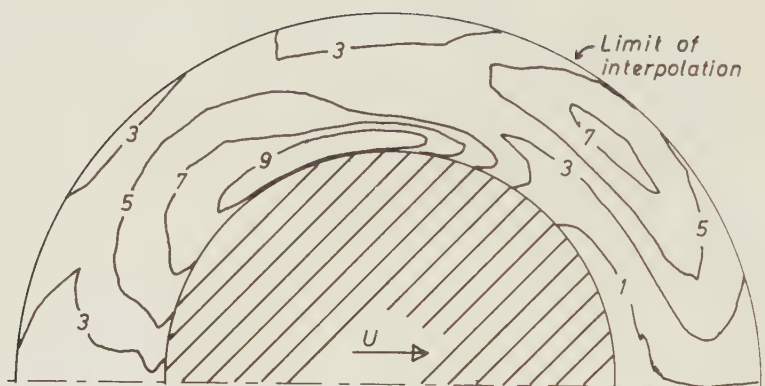
Fig. 6.18 Shear stress distributions about 5 cm circular cylinder

(a) $U = 15$ cm/s
 $H = 10$ cm

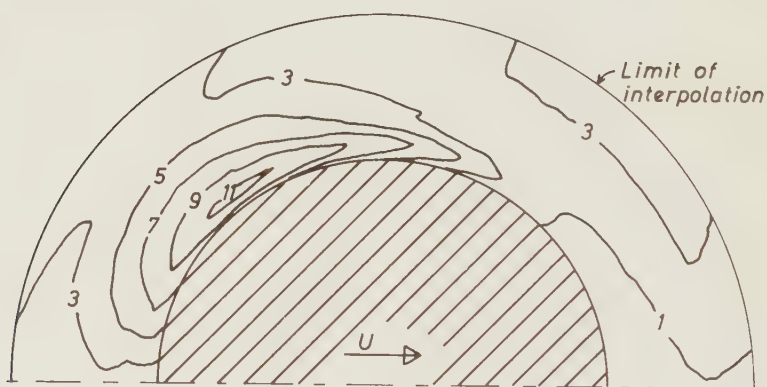
(b) $U = 30$ cm/s
 $H = 10$ cm

(c) $U = 15$ cm/s
 $H = 20$ cm

(d) $U = 30$ cm/s
 $H = 20$ cm



(a)

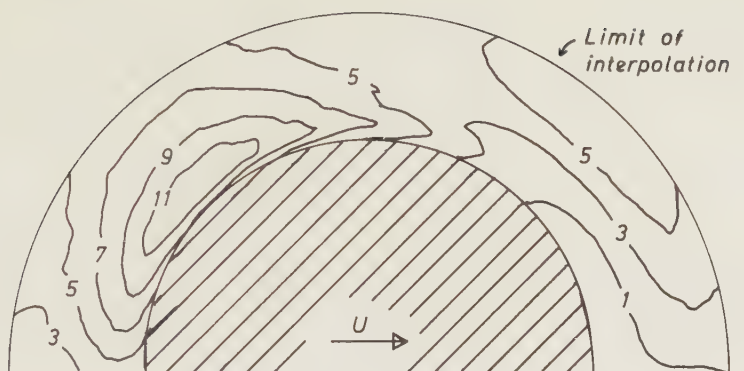


(b)

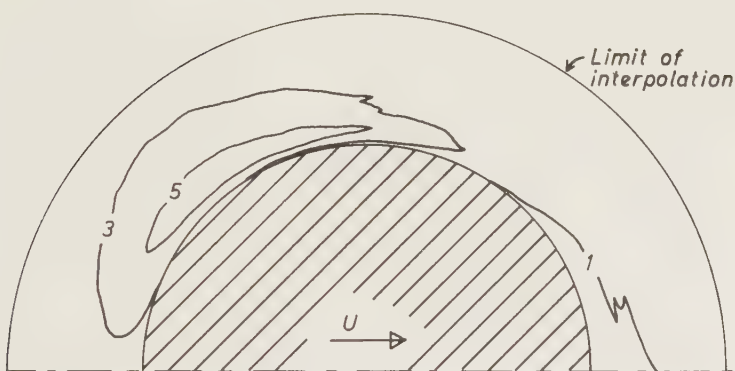
Fig. 6.19 Shear stress distributions about 7.5 cm circular cylinder

(a) $U = 30$ cm/s $H = 10$ cm

(b) $U = 15$ cm/s $H = 10$ cm



(a)



(b)

Fig. 6.20 Shear stress distributions about 7.5 cm circular cylinder

(a) $U = 30$ cm/s $H = 20$ cm

(b) $U = 15$ cm/s $H = 20$ cm

The shear stress distributions found are similar for the two cylinders tested. The horizontal scale of the shear stress field seems to be directly proportional to the diameter of the cylinder. With reservation for the bed under the edge of the wake

downstream of the cylinders, the bed area exposed to amplified shear stresses is likely to be confined to a circle, concentric with the cylinder, of radius equal to the diameter of the cylinder.

The maximum amplification of mean boundary shear was about 12 times.

For the 5 cm cylinder there is clear shift in shear stress amplification with water depth. For this cylinder the larger water depth corresponds to four times the diameter. It is consistent with the results of Chabert and Engeldinger (1956), that such large water depths may reduce the boundary shear.

For the 7.5 cm cylinder the results are somewhat dirrer. When the **velocity** is 30 cm/s a shift of water depth from 10 cm to 20 cm increases the boundary shear a little. For the actual ratios of waterdepth to cylinder diameter this is also consistent with Chabert and Engeldinger's results. The influence of water depth for $U = 15$ cm/s is more difficult to explain, but a possible explanation is that the amplification of boundary shear near the cylinder increases with increasing boundary shear of the approach flow until the latter attains some critical value. The observation of Hancu (1971) and Breusers (1972), that sediment movement near a cylindrical pier is initiated when the approach velocity is half the critical velocity, indicates that the amplification of boundary shear should be only four times for this case. Thus, as the combination of $U = 15$ cm/s and $H = 20$ cm gives the smallest boundary shear of the approach flow, it is conceivable that the maximum amplification is less in this case than for the other cases.

Of special interest is the finding that the shear stresses near the line of symmetry on the upstream side are very low. This is contrary to the assumptions of several researchers, who concluded from the shape of observed scour holes, that the tractive force should be at its maximum in this area.

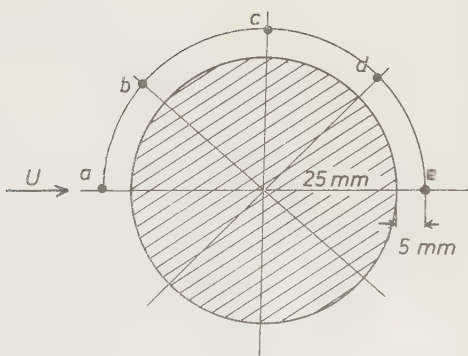
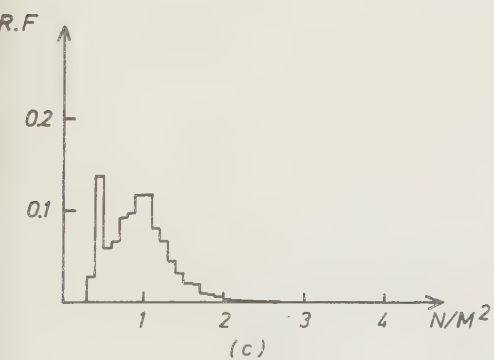
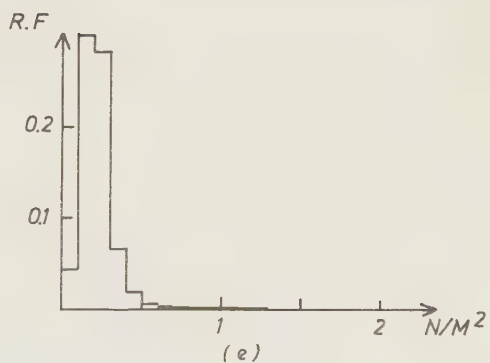
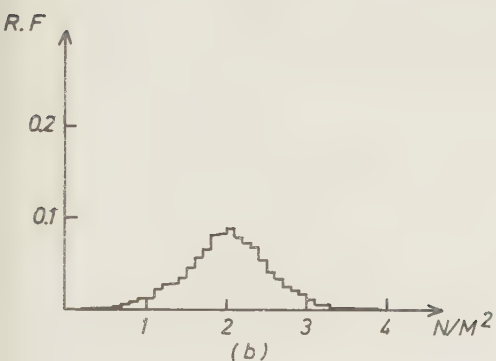
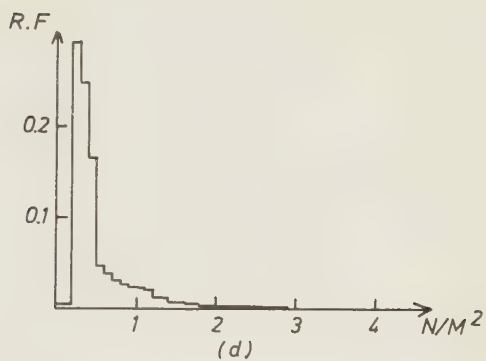
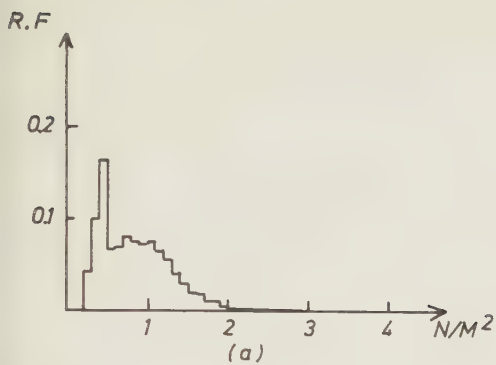


Fig. 6.21 Shear stress histograms. $U = 30$ cm/s, $B = 5$ cm, $H = 10$ cm

The temporal variation of the boundary shear was also investigated. An example of shear stress histograms is shown in Fig. 6.21.

The histograms show evident traces of intermittency, where periods of low shear with relatively low fluctuation amplitude are suddenly succeeded by periods of violent turbulent fluctuation. Thus, the fluctuating nature of the whole flow pattern is clearly revealed by the histograms.

It is also seen that the magnitude of shear stress fluctuation may be considerable. The maximum shear stress may well be about 2.5 times the mean as anticipated by Vanoni and others (1966).

6.2.2 Protecting collar

Two runs were also made where the 5 cm cylinder was supplied with a 'collar' round its foot. The outer diameter of the collar was 10 cm and its thickness was 1.25 cm near the cylinder and decreased linearly to about 0.1 cm at the outer edge. As can be seen from Fig. 6.22 it protected the bed quite efficiently from the most intense shear stress. Probably it had been even more efficient if it had been given a horse-shoe form as proposed by Engels (1894).

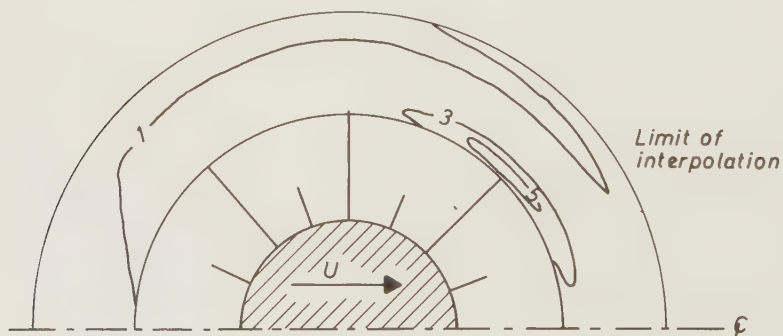


Fig. 6.22 a Shear stress distribution around 5 cm pier protected by a small collar. $U = 15$ cm/s, $H = 10$ cm

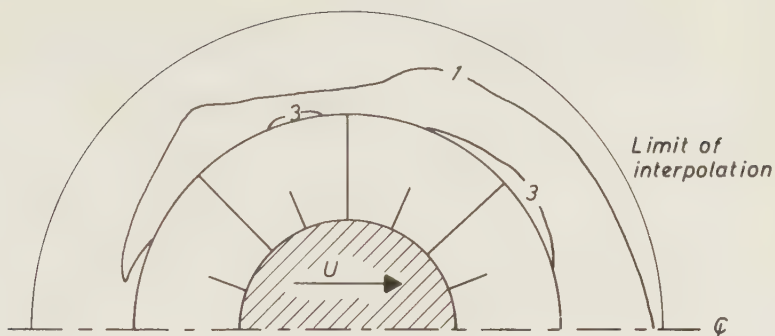


Fig. 6.22b Shear stress distribution about 5 cm cylindrical pier protected by a small collar. $U = 30 \text{ cm/s}$ $H = 20 \text{ cm}$

6.2.3 Rectangular piers

Only one cylinder of this type was tested. The cross-section was $5 \text{ cm} \times 5 \text{ cm}$ and the cylinder was tested in two positions relative to the approach flow. In one position with the upstream face normal to the undisturbed flow and in the other position with one edge facing the flow. For each position two flow conditions were tested, an approach velocity of 15 cm/s with a water depth of 10 cm , and an approach velocity of 30 cm/s with a water depth of 20 cm .

The points of measurements were positioned according to Fig. 6.23.

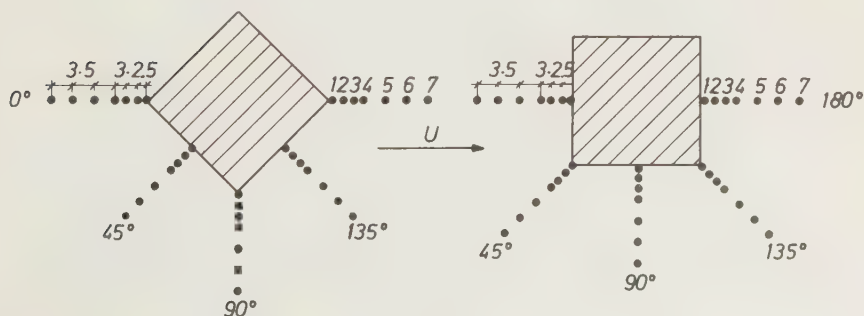


Fig. 6.23 Arrangement of points of observation about rectangular cylinders

The observed distributions of shear stresses are shown in Figs. 6.24 and 6.25.

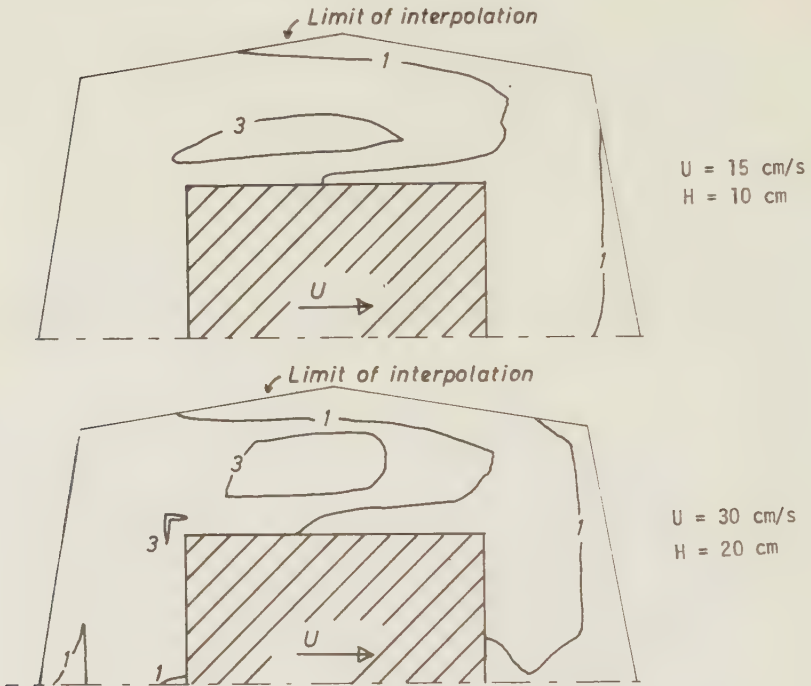


Fig. 6.24 Observed distributions of boundary shear about rectangular cylinder with one side facing the flow

From these it can be seen that the amplification of the boundary shear is similar for the two conditions of flow investigated. However, the orientation of the cylinder relative to the flow is extremely important. When the cylinder is turned with an edge facing the flow, the cylinder becomes of the pointed nose type and the shear distribution becomes similar to that of a circular cylinder, the maximum amplification of mean boundary shear being about twelve times. When the cylinder is positioned with one face normal to the flow however, the boundary shear distribution is drastically changed. The maximum amplification of the boundary shear is only about four times. This was indeed contrary

to expectations as this type of cylinder is reputed to give considerably more severe scour than circular cylinders.

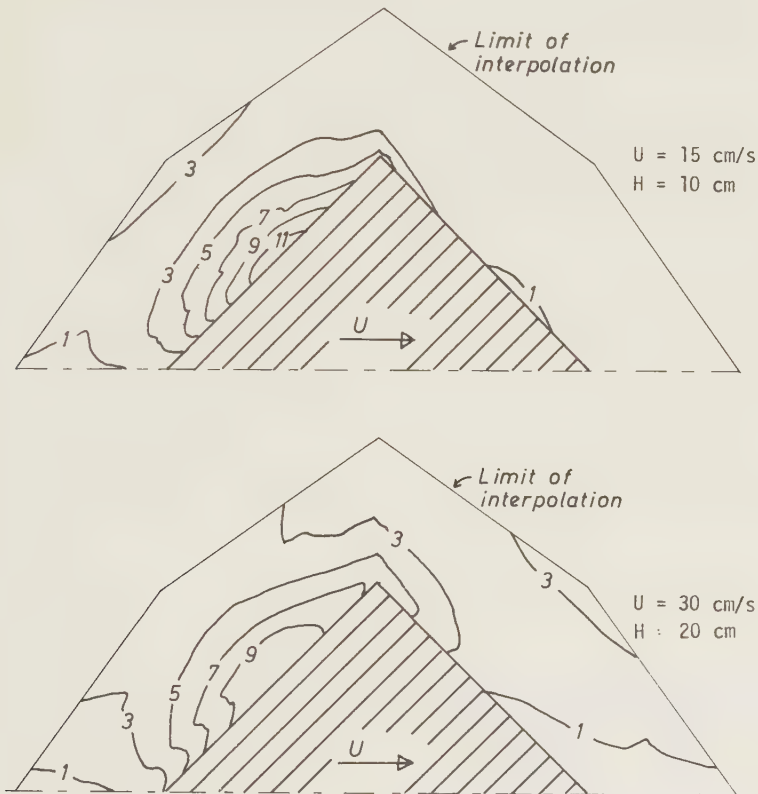


Fig. 6.25 Observed distributions of boundary shear about rectangular cylinder with one edge facing the flow

6.2.4 Pipelines

A halfway embedded pipeline was simulated with one half of a 3.9 cm diameter longitudinally cut bar, which was laid on the flume bottom at right angles to the flow. The length of the bar was equal to the width of the flume. Two different conditions

of flow were tested, one with the velocity of 15 cm/s at 10 cm water depth, and the other with a velocity of 30 cm/s at a water depth of 20 cm. As can be seen from Figs. 6.26 and 6.27 the pipeline tended to reduce the mean boundary shear in the vicinity of the pipe.

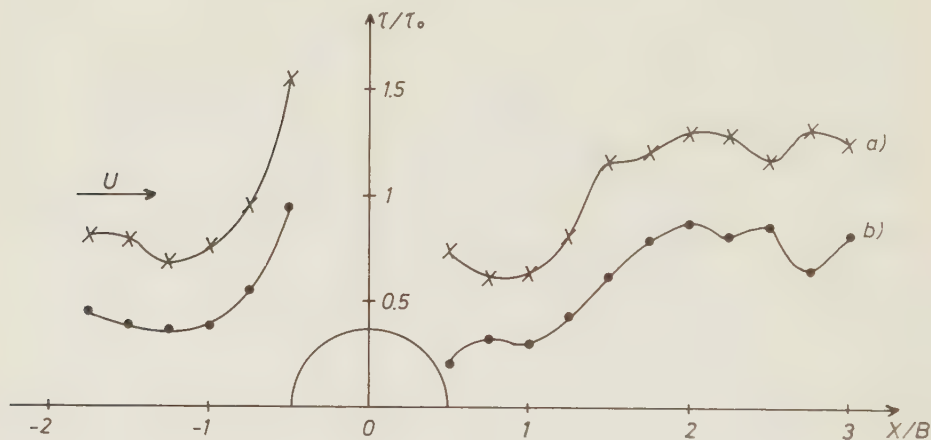


Fig. 6.26 Boundary shear near pipelines

(a) $U = 30$ cm/s $H = 20$ cm
(b) $U = 15$ cm/s $H = 10$ cm

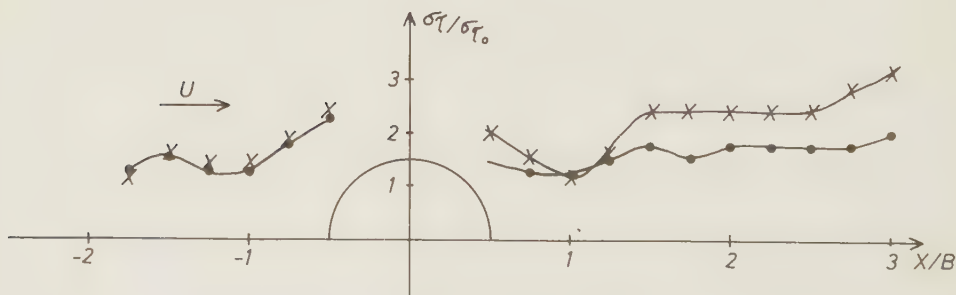


Fig. 6.27 Turbulent fluctuation of boundary shear near pipelines

Though the amplitude of the turbulent boundary shear fluctuation is increased in the surrounding of the pipe, it was concluded that boundary shear stresses are not likely to play the most important part in the undermining of pipelines.

6.3 Pressure distribution about cylinders and pipes

Earlier studies, both own and others, have clearly pointed out the risks connected with neglect of pressure forces in the establishment of scour criteria. Therefore, an attempt was made to study, at least qualitatively, the pressure forces involved in flows about cylinders and pipes.

For these studies a test series involving a semimovable and pervious bed was designed. A horizontal concrete flume of width 2 m and of length 39 m was used for the experiments. In the flume, a 20 cm high sand bank was erected. The length of the horizontal crest of the bank was about 4 m and it was provided with upstream and downstream slopes of 1:10, Fig. 6.28

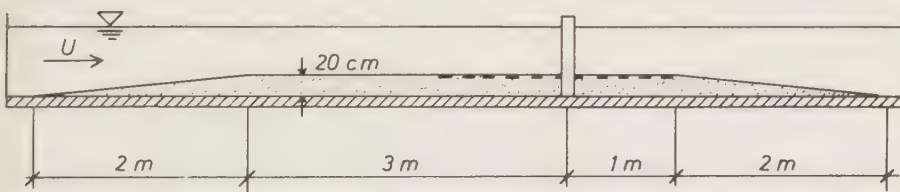


Fig. 6.28 Experimental setup for pressure studies.
The dotted line signifies pointed sand layer.

The bed surface within a distance of 1 m from the cylinder or pipe was painted with a thin layer of PVC paint to prevent scour to go below this level. Before the surface was painted, pressure tapings were provided around the obstacle to be tested. The pressure taps consisted of brass tubes, with

their openings protected by pieces of cloth. The openings were placed flush with the surface to be painted.

The brass tubes were connected to a pressure transducer, outside the flume, by means of plastic tubes, which were chosen after comparison with brass tubes, where it was found that, to the precision of the pressure transducer, the signal from the plastic tube was equivalent to the signal from the brass pipe.

The pressure transducers used were two integrated piezoelectric circuits with vacuum reference cells, which were put in circuits giving either the individual momentaneous pressure or the pressure difference. The specified precision was ± 10 mm H_2O for absolute levels and the resolution was 1 mm H_2O which means that the differences of mean pressures in one specific run could be determined with a precision of about ± 1 mm H_2O . Unfortunately one of the transducers did not meet the specifications and therefore the momentaneous pressure differences could not be measured with any meaningful precision. However, the transducers were judged to be reliable enough to give meaningful cross-correlation values.

When an experiment was started, sand was dumped upstream of the bank to serve as a sediment source. As the flows were strong enough to induce bed load movement, ripples were formed. These covered the crest of the bank, except near the cylinder, where the painted layer was kept free from sand by the secondary flows.

Only one type of sand was used. This was a quartz-comprised sea sand of mean diameter 0.11 mm. The gradation curve is shown in Fig. 6.29.

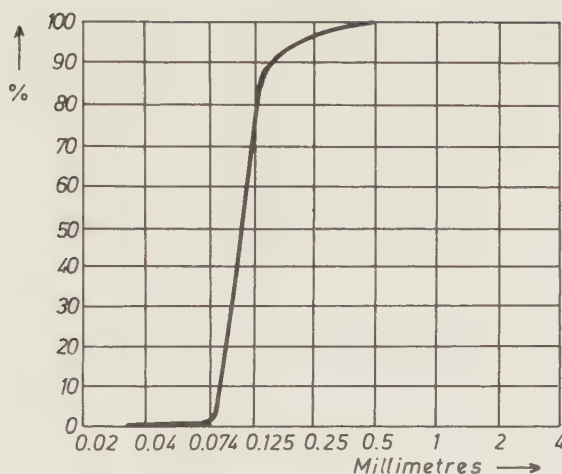


Fig. 6.29 Gradation curve for the sand used in the experiment

To determine the effect of the paint layer, infiltrometer tests were made with natural sand and with sand covered by a paint coating. It was found that the coating reduced the infiltration velocity by a factor of about five.

Though the paint caused a considerable anisotropy, it was judged that the observations on pressure distribution and variation should be qualitatively correct.

6.3.1 Circular cylinders

The pressure taps were located on the circumferences of two circles, concentric with the cylinder. The positioning and numbering of the taps is illustrated in Fig. 6.30.

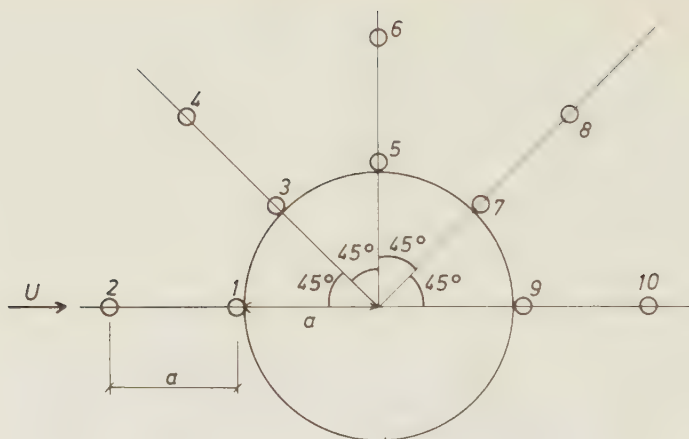


Fig. 6.30 Arrangement and numbering of pressure taps about circular cylinder

The pressure distribution along the inner circle was found to correspond very well to the pressure distribution in two-dimensional flow if the pressure is related to the stagnation pressure of the mean velocity of the approach flow. This may seem surprising, considering that the approach velocity near the bed is considerably lower than the mean velocity. However, the phenomenon is to be ascribed to the effects of the secondary flow and shows what power this flow has to redistribute fluid forces of the approach flow.

The most spectacular difference between two-dimensional and three-dimensional pressure distributions seems to be their dependence on Reynolds' number. This point is illustrated in Figs. 6.31 and 6.32 where observed pressures are compared to pressures observed in two-dimensional flows at different Reynolds' numbers.

The pressures are represented by the dimensionless pressure coefficient

$$C_p = \frac{p - p_\infty}{\frac{1}{2} \rho u^2}$$

where

p_∞ = the static pressure of the undisturbed flow

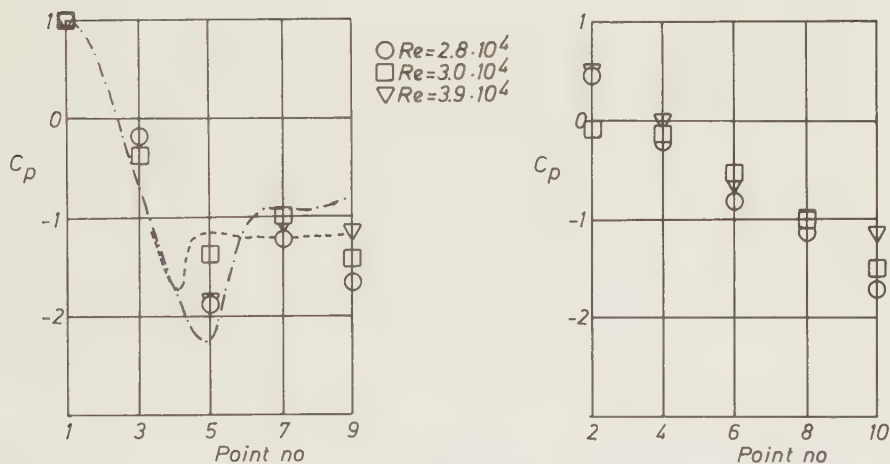


Fig. 6.31 Variation of mean pressure about circular cylinders. The lines represent two-dimensional pressure distributions as measured by Achenbach (1968)

- - - - $Re = 10^5$;
 - · - · $Re = 2.6 \cdot 10^5$

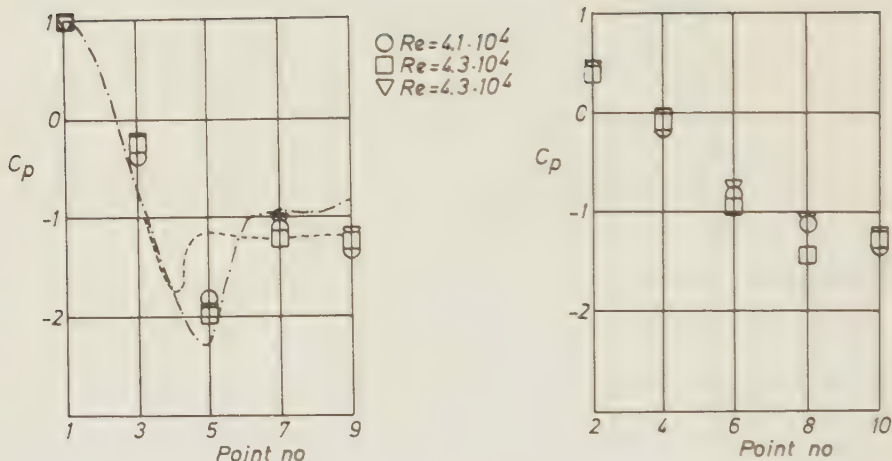


Fig. 6.32 Variation of mean pressure about circular cylinders. The lines represent two-dimensional pressure distributions as measured by Achenbach (1968)

- - - - $Re = 10^5$
 - · - · - $Re = 2.6 \cdot 10^5$

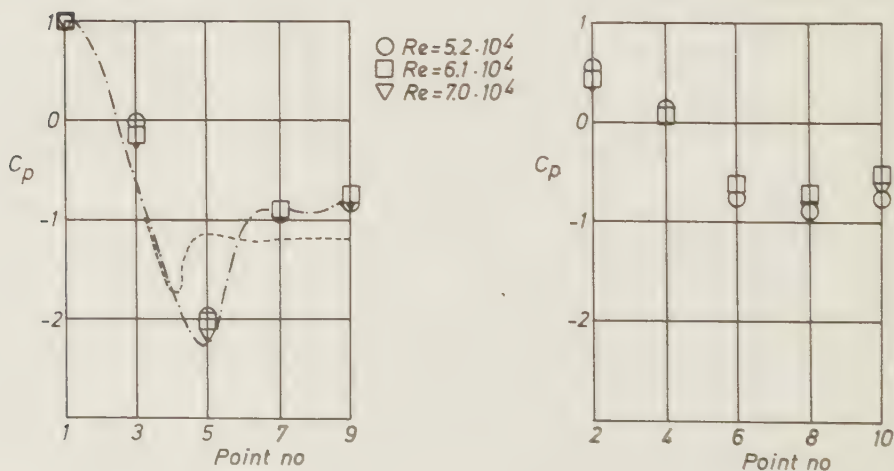


Fig. 6.33 Variation of mean pressure about circular cylinders. The lines represent two-dimensional pressure distributions as measured by Achenbach (1968)

- - - - $Re = 10^5$
 - · - · - $Re = 2.6 \cdot 10^5$

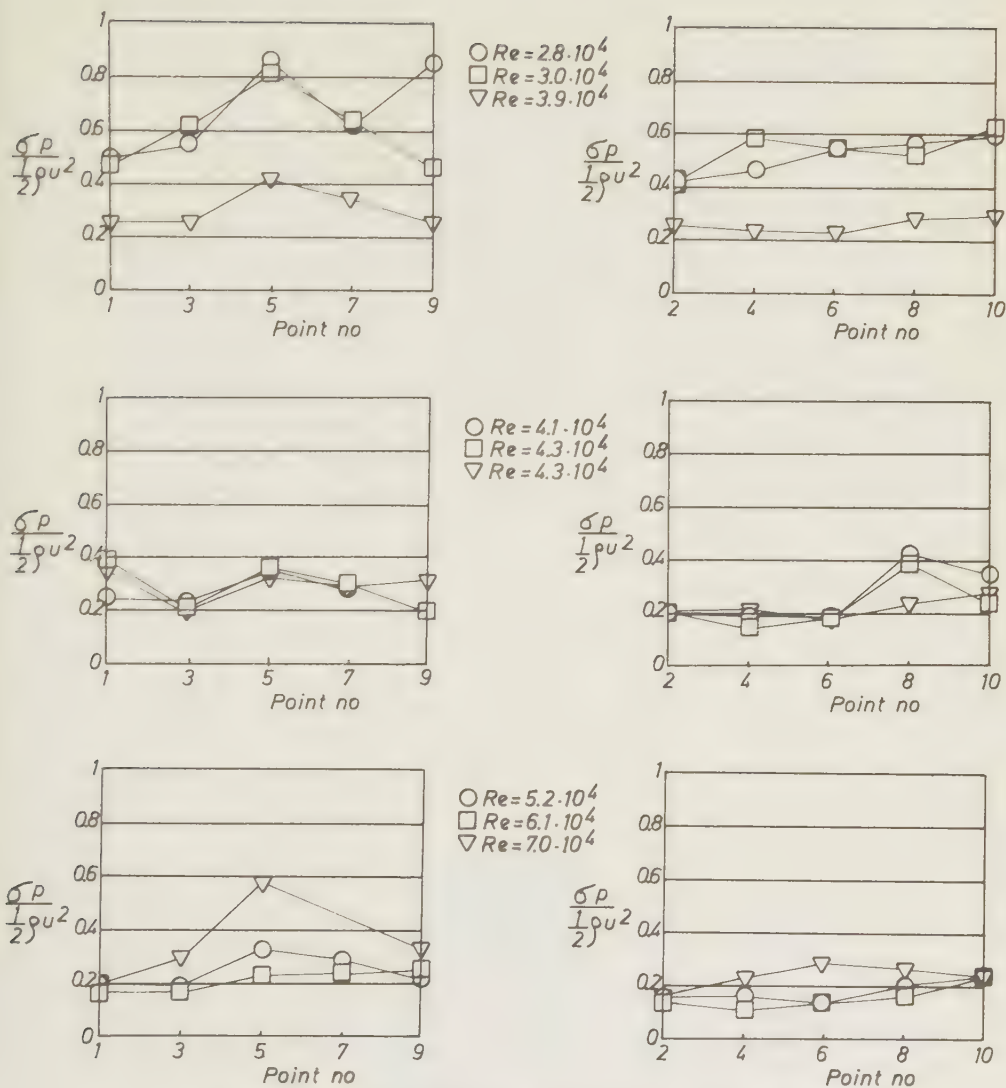


Fig. 6.34 Turbulent pressure fluctuations about circular cylinder

Thus, it seems that the pressure distribution for a 3-dimensional flow corresponds to the pressure distribution of a 2-dimensional flow of a Reynolds' number, which is about 5 times as large. The Reynolds' number is seen to affect mostly the position of the point of separation and the pressure of the wake.

For the outer points the pressure gradients are smaller. The dimensionless pressures, c_p , diminish with increasing Reynolds' number, Fig. 6.33.

The turbulent fluctuations of the pressure may be considerable. Upstream of the wake, the range of pressure fluctuation is about $\frac{1}{2} \rho U^2$ and in the wake region, the range of fluctuations is about $\frac{1}{2} \rho U^2$. The distribution of the pressure standard deviation is shown in Fig. 6.34.

6.3.2 Rectangular cylinders

Around this cylinder, the pressure taps were arranged according to Fig. 6.35.

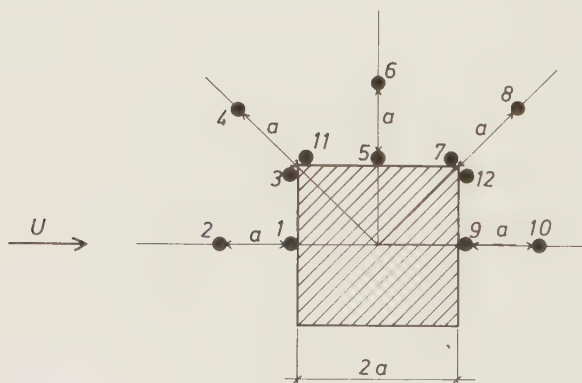


Fig. 6.35 Pressure taps about rectangular cylinder

The measurements showed that the pressure variation near the cylinder is very much concentrated to the upstream corner. The pressure distribution over the outer points is similar but smoother, Fig. 6.36.

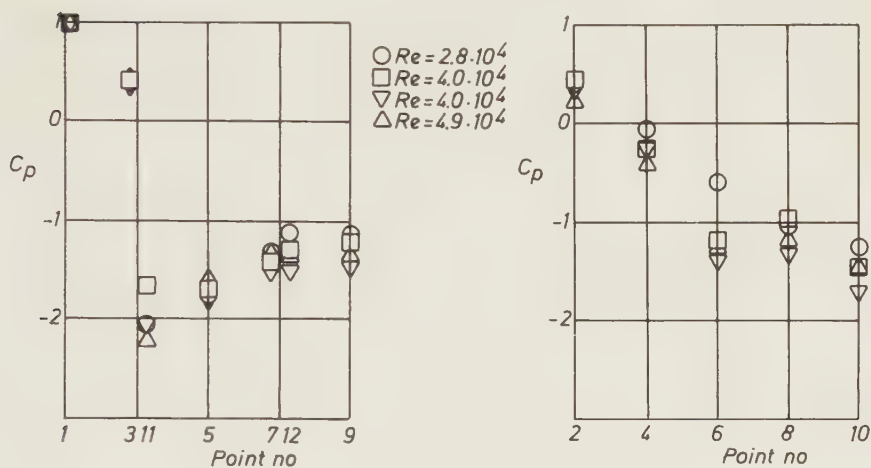


Fig. 6.36 Distribution of mean pressure around rectangular cylinder

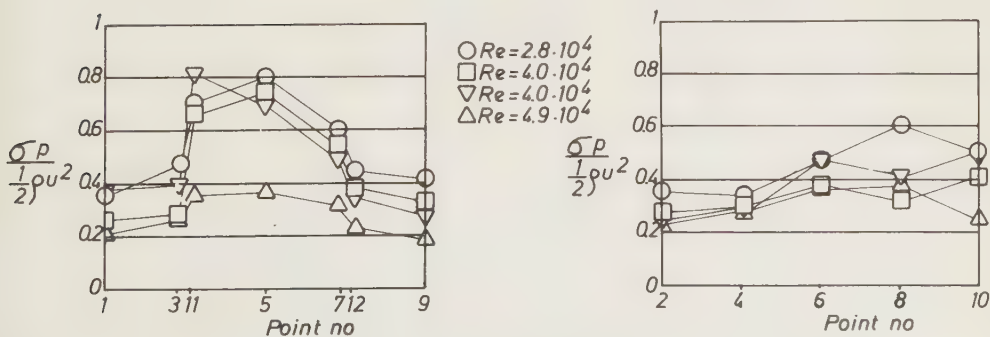


Fig. 6.37 Turbulent pressure fluctuation around rectangular cylinder

On the upstream side, the pressure fluctuations have a range of approximately $\frac{1}{4} \rho U^2$. In the wake flow, along the sides of the pier the fluctuations are important, with a range of about $\frac{3}{2} \rho U^2$. Thus, the momentaneous pressure drop over the upstream corner may be considerable, of order $2 \rho U^2$.

An attempt was made to study the pressure characteristics in a scour hole. Therefore the bed was allowed to scour for a couple of hours before the pressure taps were installed and the bed surface was painted. The openings of the pressure taps were flush with the surface and the horizontal spacing between taps was as for the plane bed. However, this time also three subsurface pressure taps were installed. These were placed between taps number 6 and 8 and respectively 1 cm, 2 cm, and 3 cm below bed surface.

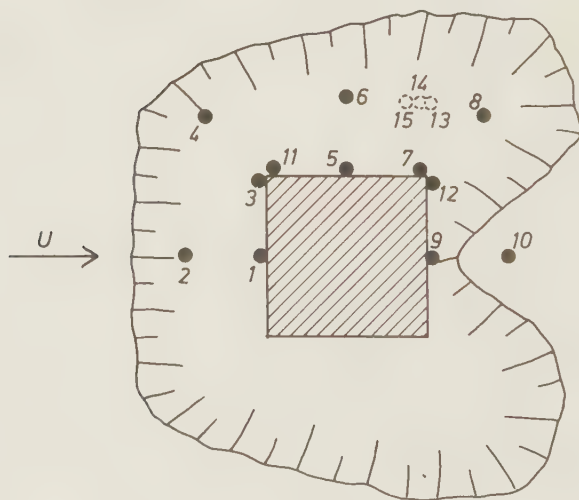


Fig. 6.38 Pressure taps in scour hole

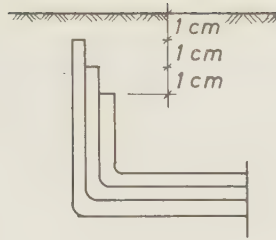


Fig. 6.39 Arrangement of sub-surface pressure taps

The flow in the scour hole was in fact less chaotic than around the cylinder on the plane bed. The pressure gradients were reduced and the wake at the upstream corner was much smaller. A consequence of this is the pressure drop at the downstream corner which was not observed on the plane bed. This indicated a flow towards the back side, a flow which was manifested by a sediment accumulation behind the cylinder. The observed characteristics of the pressure are shown in Figs 6.40 and 6.41.

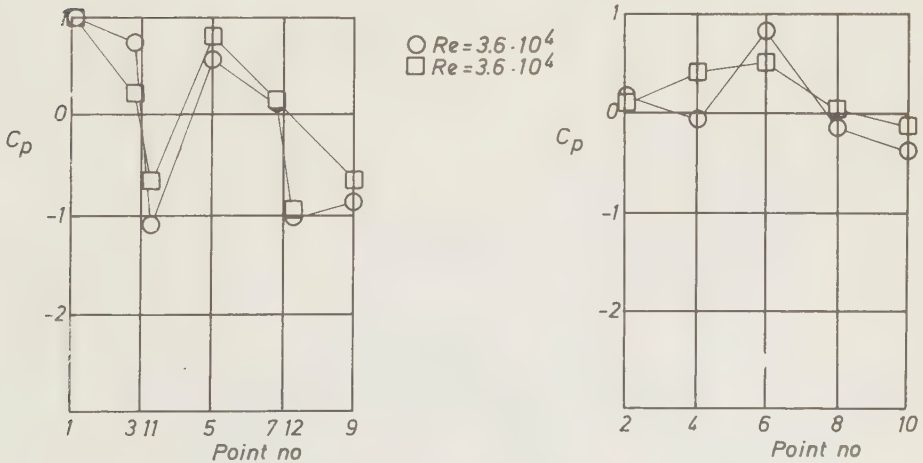


Fig. 6.40 Distribution of mean pressure in scour hole

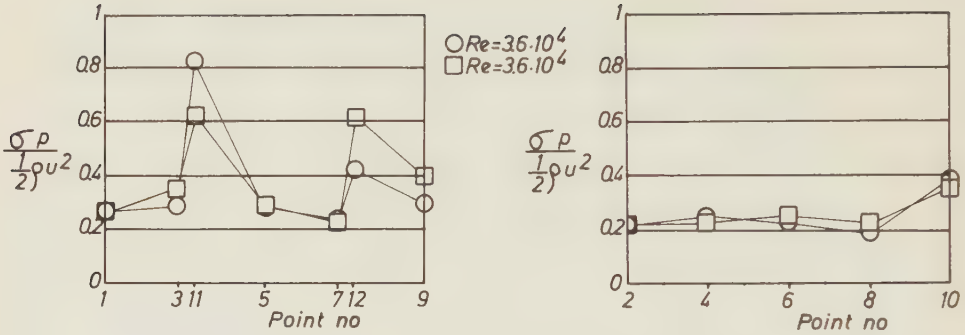


Fig. 6.41 Turbulent pressure fluctuations in scour hole

The subsoil pressure taps showed that the pressure fluctuations protrude into the bed. The values found are not representative because of the painted surface, but they indicate that pressure fluctuations can indeed influence the stability of the bed sand. The values of the mean pressures and fluctuations observed were.

Point	6	8	13	14	15
C_p	0.65	- 0.07	- 0.07	- 0.11	- 0.16
$C_{p'}$	0.22	0.20	0.07	0.07	0.07

Table 6.1 Observed pressure fluctuations on and below bed surface

6.3.3 Rectangular pier turned 45°

The numbers and locations of the pressure taps are shown in Fig. 6.42

The distribution of pressure and pressure fluctuation is summarized in Figs. 6.43 and 6.44.

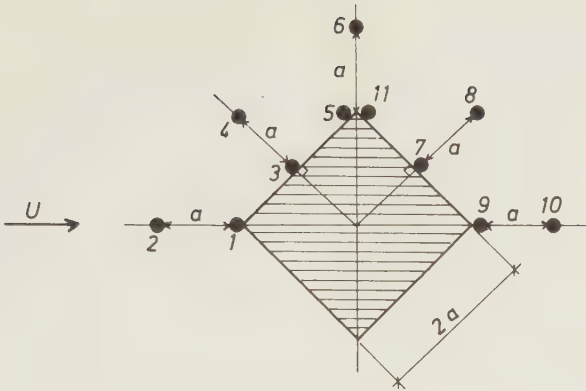


Fig. 6.42 Spacing of pressure taps around pointed nose cylinder

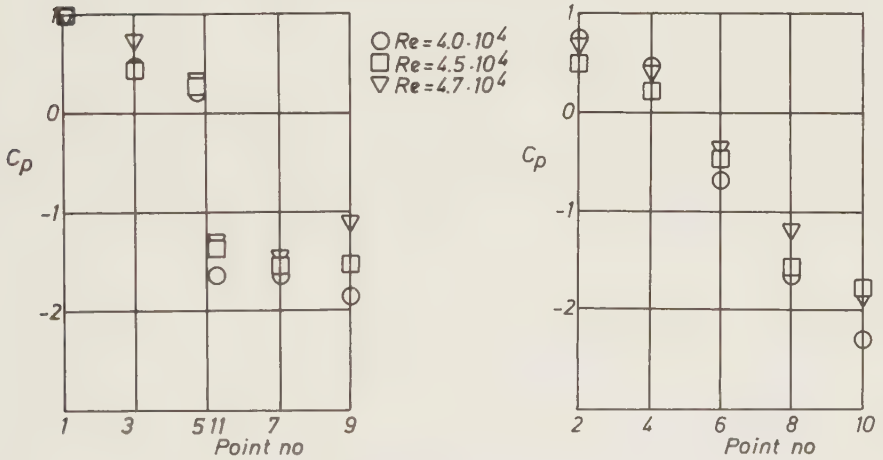


Fig. 6.43 Distribution of mean pressure about sharpnosed pier

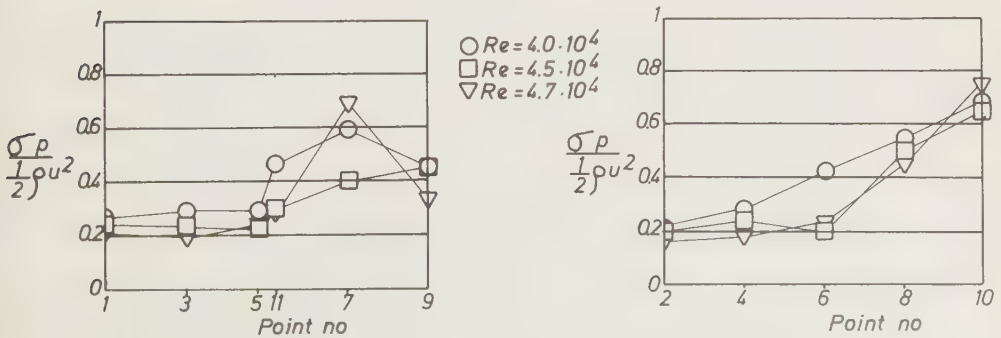


Fig. 6.44 Turbulent pressure fluctuations about sharpnosed pier

Also around this cylinder the pressure drop is concentrated to the region of separation. Here the mean pressure drop is of order $\frac{3}{4} \rho U^2$. The range of fluctuation is about $\frac{1}{4} \rho U^2$ upstream and $\frac{1}{2} \rho U^2$ downstream of the point of separation. If the fluctuations upstream respectively downstream may be considered independent, the momentaneous pressure drop over the region of separation may reach a value of about $\frac{7}{4} \rho U^2$.

6.3.4 Pipelines

Two series of tests were performed. In one the pipe was laid on the originally plane bed, and in the other, the pipe was half-way embedded in the sand.

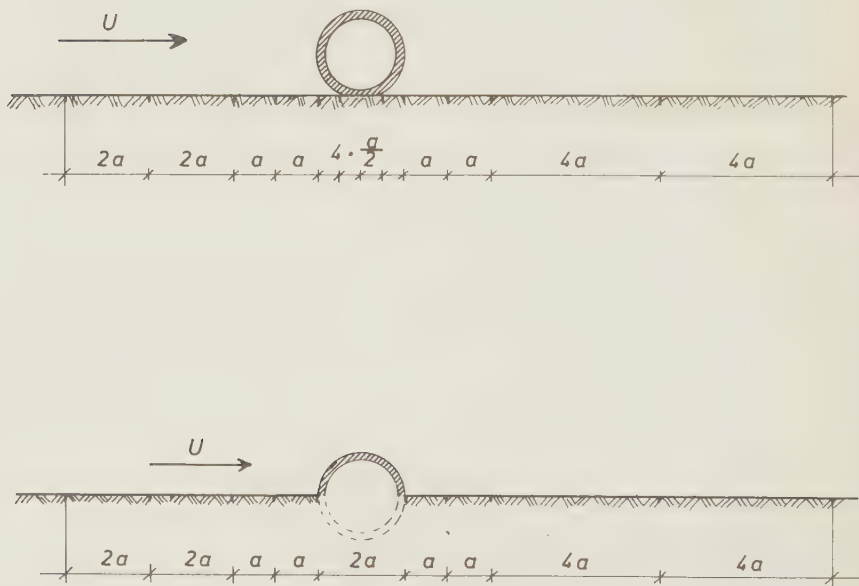


Fig. 6.45 Pressure taps near pipelines

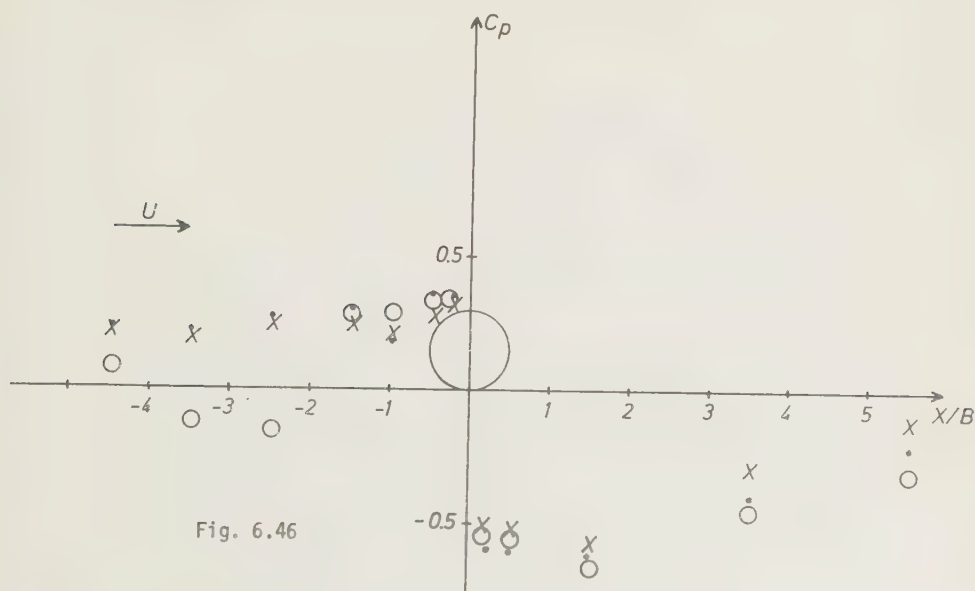
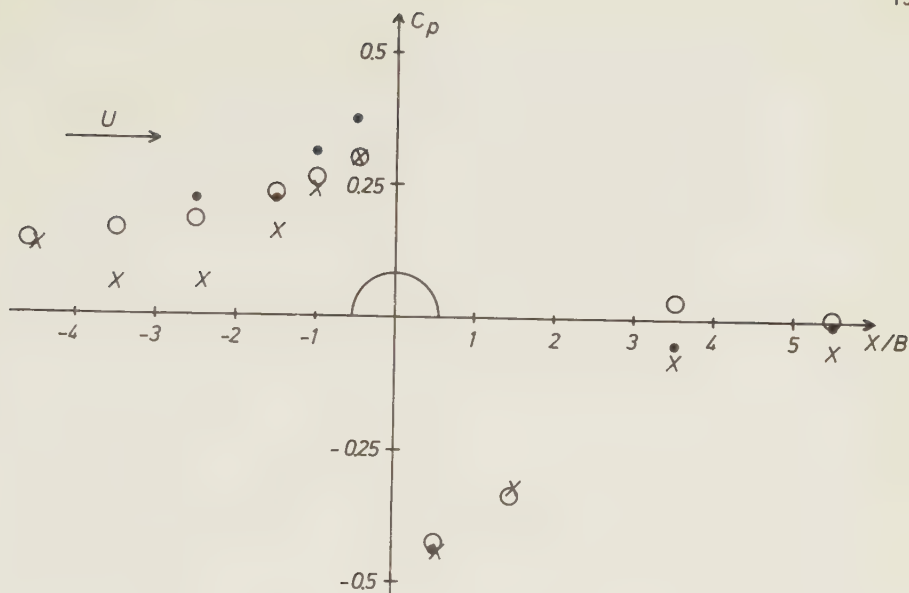


Fig. 6.46

Fig. 6.46 Distribution of mean pressure near pipelines

The pipe used was a 11 cm perspex pipe and as the depths of flows were between 23 and 33 cm, the contraction of the flow section over the pipe was considerable. It was found that for these conditions the mean velocity of the most contracted section U_p was a better reference velocity than the mean velocity U of the approach flow. Thus the observed dimensionless pressures C_p were related to U_p rather than U . A plot of the results is shown in Figs. 6.46 and 6.47.

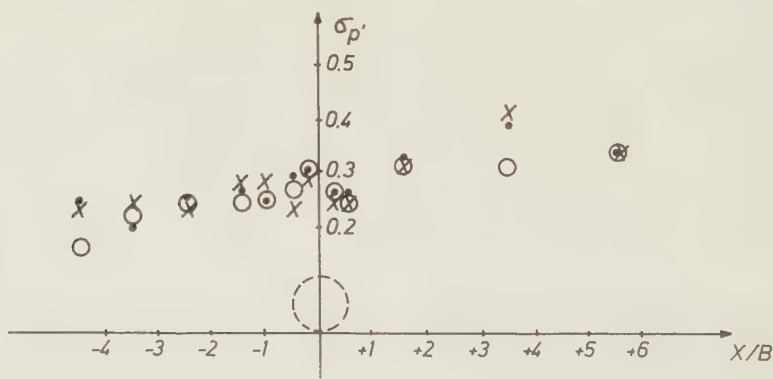


Fig. 6.47 Turbulent pressure fluctuations about pipelines

• $U = 31.5$ cm/s $H = 25.4$ cm
 x $U = 43.9$ cm/s $H = 30.2$ cm
 o $U = 31.6$ cm/s $H = 32.4$ cm

The mean pressure drop over the pipe is thus seen to be about $\frac{1}{2} \rho U_p^2$ for the pipe resting on the bottom and about $\frac{2}{5} \rho U_p^2$ for the halfway embedded pipe. The range of variation near the pipe was found to be approximately equal to the mean pressure. Thus, the maximum momentaneous pressure differences between the upstream and downstream sides of the pipe are about ρU_p^2 for a pipe resting on the bed and about $\frac{3}{4} \rho U_p^2$ for a half-way embedded pipe. The experiments also showed that the amplitude of the pressure fluctuations near the point of reattachment may be considerable.

6.4 Statistical analysis of pressure fluctuations

The statistical properties of the pressure fluctuations are influencing the stability of the bed sand in several ways.

The probability density function determines what percentage of the pressure fluctuations that will be of dangerous sizes for a given mean amplitude and a given mean pressure.

The autocorrelation function gives the characteristic time scale of the fluctuations, which is an important parameter for the seepage rate.

Finally, the cross-correlation function reveals to what extent the fluctuating pressures at neighbouring points are independent, a fact of prime importance for the momentaneous magnitudes of the horizontal pressure gradients.

Only recordings from the rectangular cylinder, with its front normal to the approach flow, were used for the statistical analysis. The numbering of the pressure taps is shown in Fig. 6.35.

6.4.1 Probability density function

The pressure fluctuations were found to fit the Gaussian distribution quite well, with exception for a small skewness. It is interesting to note that the observed probability densities do not show any traces of any periodic component. In Fig. 6.48 some observed probability density functions are shown together with the Gaussian probability density function.

The kurtosis, or peakedness, was computed for the observed distributions. For all points it was found to be close to the kurtosis of the Gaussian distribution.

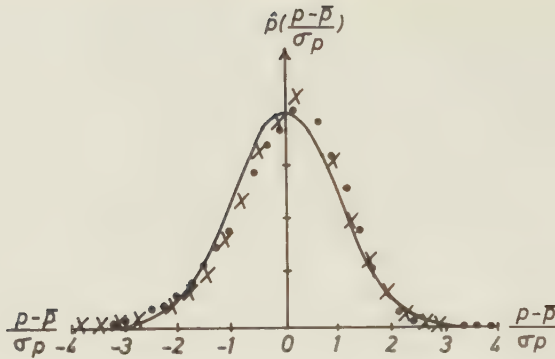


Fig. 6.48 Examples of observed probability density functions
 -- Gaussian distribution = point 3
 • point 11
 x point 10

6.4.2. Auto correlation function

The auto correlation functions show traces of periodicity. This is especially marked at the upstream corner, points 3 and 11. The period of the fluctuations in the wake besides the cylinder is about 3 seconds, which gives a Strouhal number of $Sh = 1/3 \cdot 0.11/0.3 \approx 0.12$. This value is about half the value for a circular cylinder of equal width, in two-dimensional flow. Thus, the time scale is about twice the time-scale which could have been expected. Some of the observed auto correlation functions are shown in Fig. 6.49. At point 3 however, the period seems to be 1.5 s, which corresponds to the time scale for a circular cylinder of equal width in two-dimensional flow.

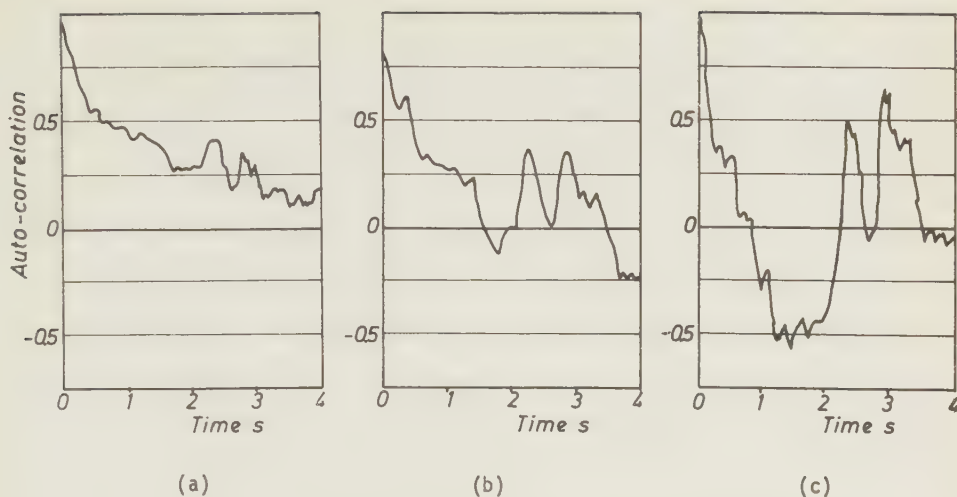


Fig. 6.49 Examples of observed auto correlation functions
 (a) point 9
 (b) point 5
 (c) point 11

6.4.3 Cross correlation functions

Between most points the cross correlation is negligible. However, between points 3 and 11 there is some correlation, which was to expect since they both are exposed to periodic fluctuations. Of most interest is the cross correlation for zero delay as this affects the magnitude of the momentaneous horizontal pressure gradients. These correlation coefficients are generally small and thus the maximum fluctuating pressure difference between two points may well be given by a maximum value of the positive pressure fluctuation in one point together with a maximum value of the negative fluctuation in the other point. Some observed cross correlation functions are shown in Fig. 6.50.

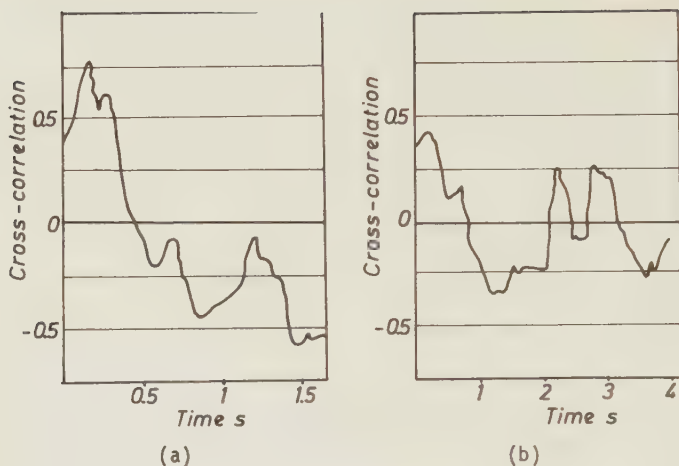


Fig. 6.50 Examples of observed cross correlation functions
 (a) points 3 and 11
 (b) points 11 and 6

6.5 Sediment movement

When a thin, 1 cm, plane layer of loose sediment was laid on the model bank, sediment movement could be studied.

For circular cylinders it was found that for velocities, slightly higher than the velocity causing incipient movement about the cylinder, sediment movement is most pronounced downstream of the cylinder, at the edge of the wake. It seemed that water flowing under the trailing vortices, into the low pressure zone of the wake, dug into the bed and transported sediment into the wake region. As the velocity of flow was increased, the region of most intense sediment motion shifted to the upstream nose of the cylinder. Thus, here is another indication of a shift in flow structure for relatively low Reynolds numbers. As has been seen earlier the relative pressure in the wake is at its lowest for small Reynolds numbers and as shown by the shear stress measure-

ments, the relative strength of the horse-shoe vortex seems to depend on the Reynolds number if this is relatively small.

In the wake region strong instationary vortices were observed. These had an important capacity to move the bed sediment. However, these vortices moved quite randomly and therefore most of the sediment was only moved about in the wake region, though some sediment was moved to the rear of the cylinder and a small portion of the sediment was lost to the surrounding flow.

If the erosion process was allowed to continue for sufficiently long time, the area near the cylinder was scoured down to the painted layer and the shape of the sedimentfree area attained some equilibrium shape, Fig. 6.51. This shape seemed to be given by the dividing boundary streamline. This means that there is no bed load transport into this area. Of course some sediment is fed into the scoured area through sliding from the neighbouring ripples.

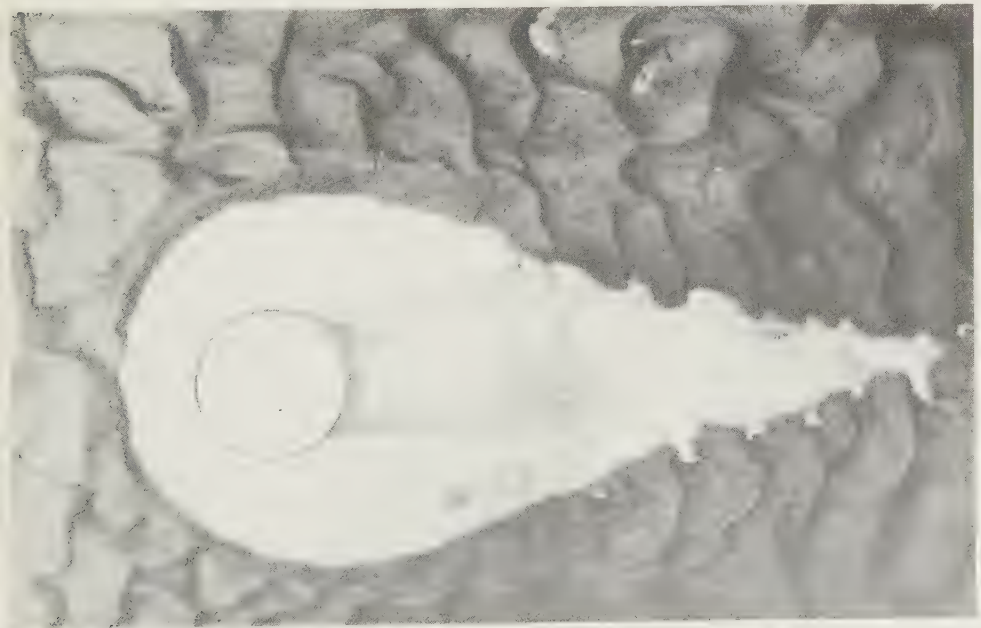


Fig. 6.51 Scour around circular cylinder

A test series with $U = 35$ cm/s and $H = 25$ cm was performed to investigate the scouring capacity within the scoured region. First an equilibrium scour shape was allowed to form then the scour 'hole' was filled with sand of diameter 0.5-1 mm, compared to a mean diameter of 0.11 mm for the surrounding bed. When the flow, was reestablished the coarser sediment eventually was scoured completely away and the scour area re-attained its equilibrium shape.

Next the scour hole was filled with 1 - 2 mm sediment. The result was the same as for the 0.5 - 1 mm sediment, not a single grain was left and the original equilibrium shape was again attained. However, the scour hole developed considerably slower than for the finer sediments.

Then a 2 - 4 mm material was tried. This time there was a considerable reduction of the scoured area and the equilibrium was attained much slower than for the finer sediments.

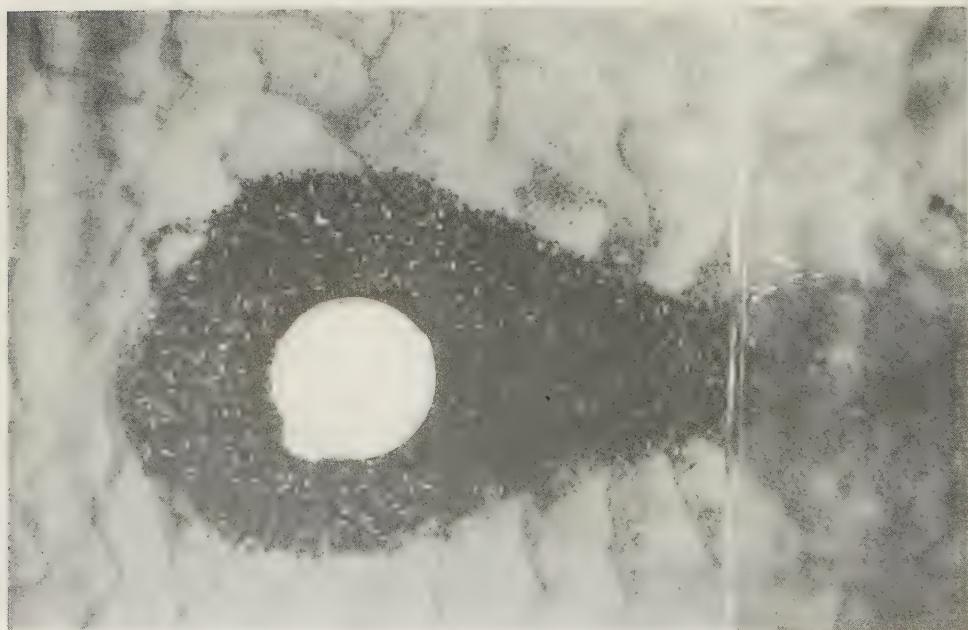


Fig. 6.52 Scour pattern after 3 minutes 2 - 4 mm sand

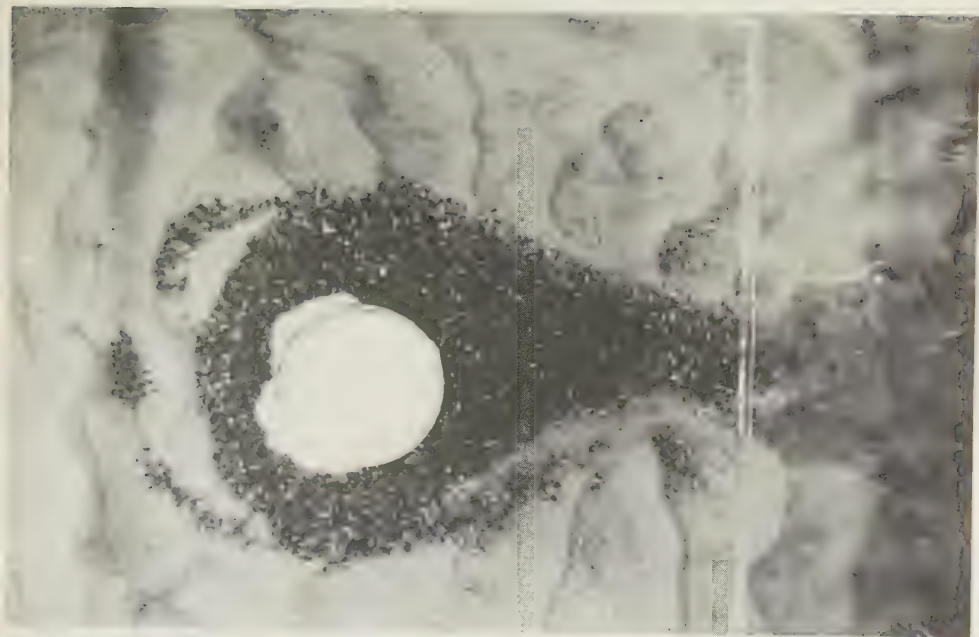


Fig. 6.53 Scour pattern after 30 minutes 2 - 4 mm sand

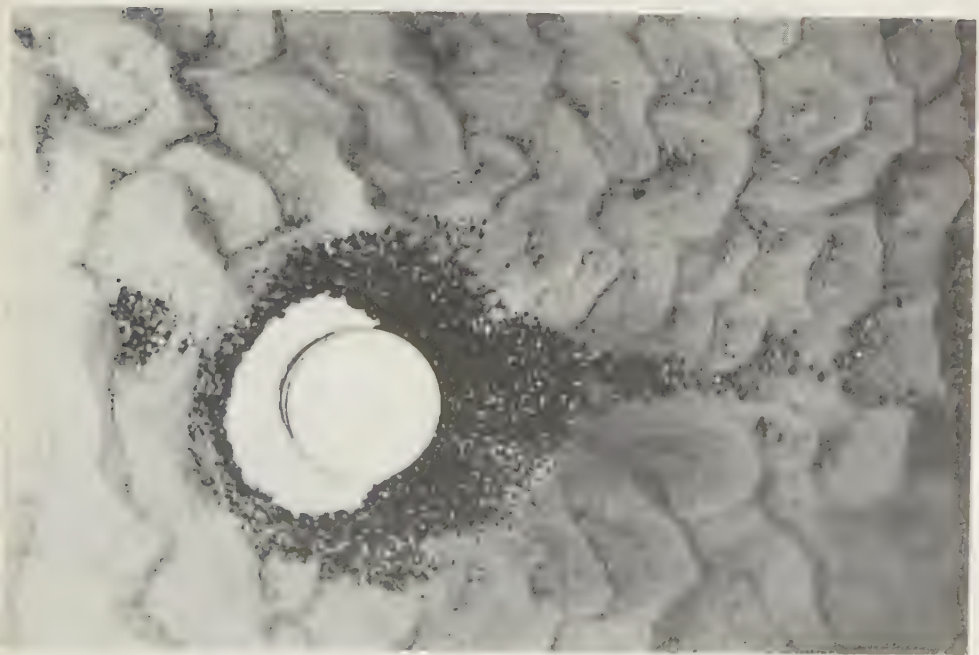


Fig. 6.54 Final scour pattern 2 - 4 mm 4 hours

Finally this smaller scour hole was filled with 4 - 8 mm gravel. This time the bed was stable. Only minor depressions of the bed were observed. The bed shape after a 4 hour run is shown in Fig. 6.55.

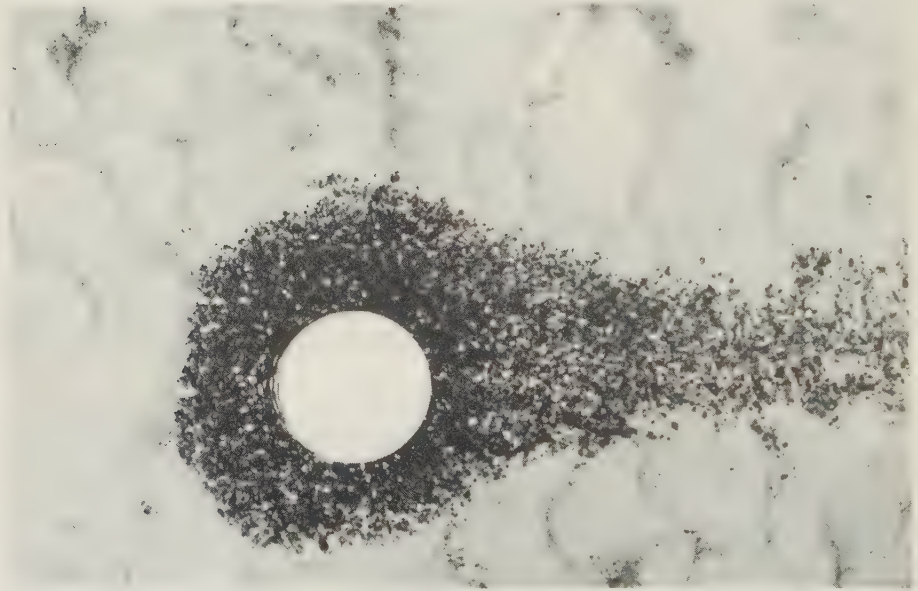


Fig. 6.55 Protected cylinder after a 4 hour run

It is interesting to note the increased amount of fine sediment in the wake region downstream of the cylinder. This indicates that though the gravel layer has a thickness of only 1 - 2 cm it has a noticeable influence on the flow structure in the wake region. This fact stresses the warnings of preceding investigators, that the vertical position of a protective layer must be carefully considered.

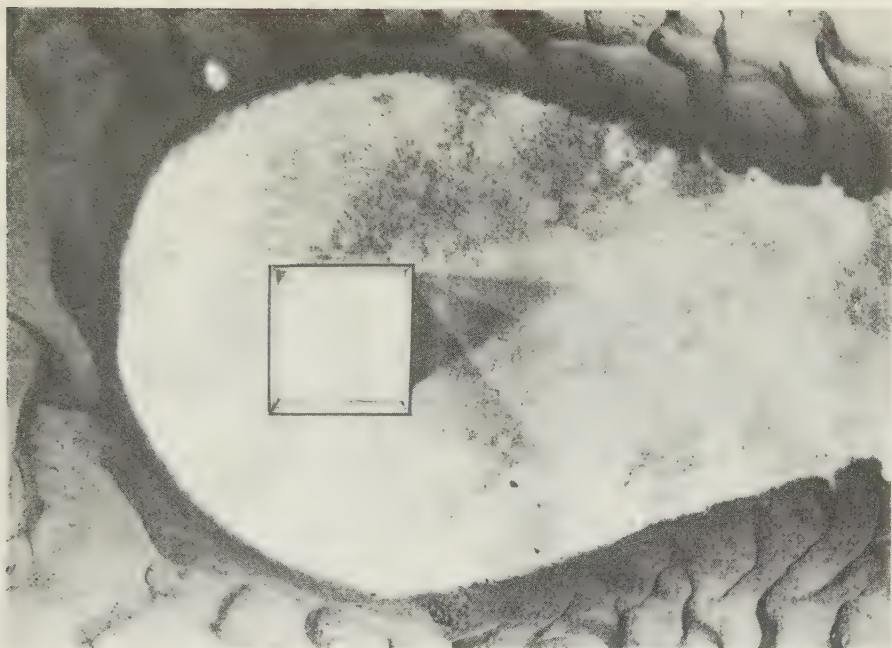


Fig. 6.56 Original equilibrium scour shape

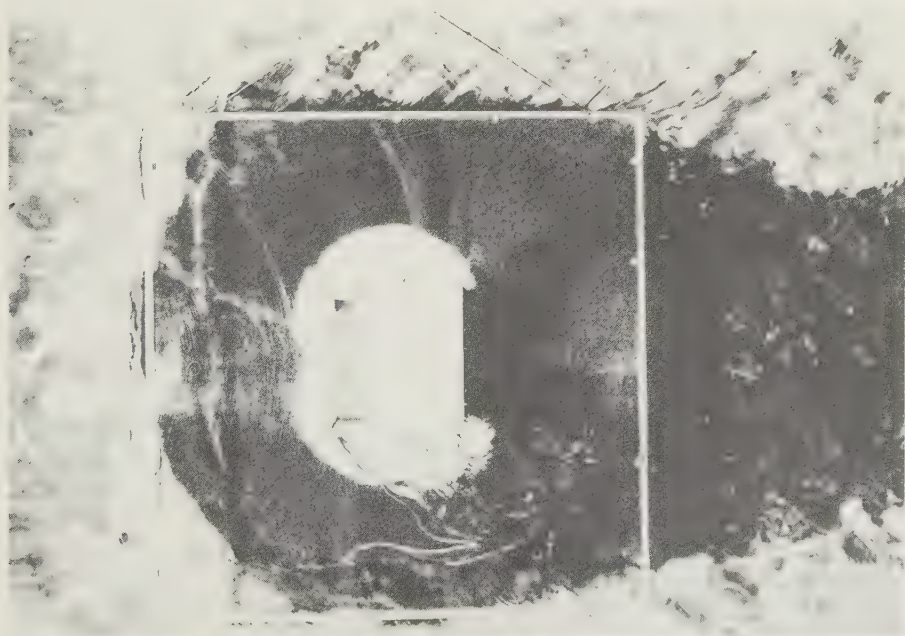


Fig. 6.57 Scour pattern after 5 min 2 - 4 mm



Fig. 6.58 Scour pattern 4 hrs 2 - 4 mm

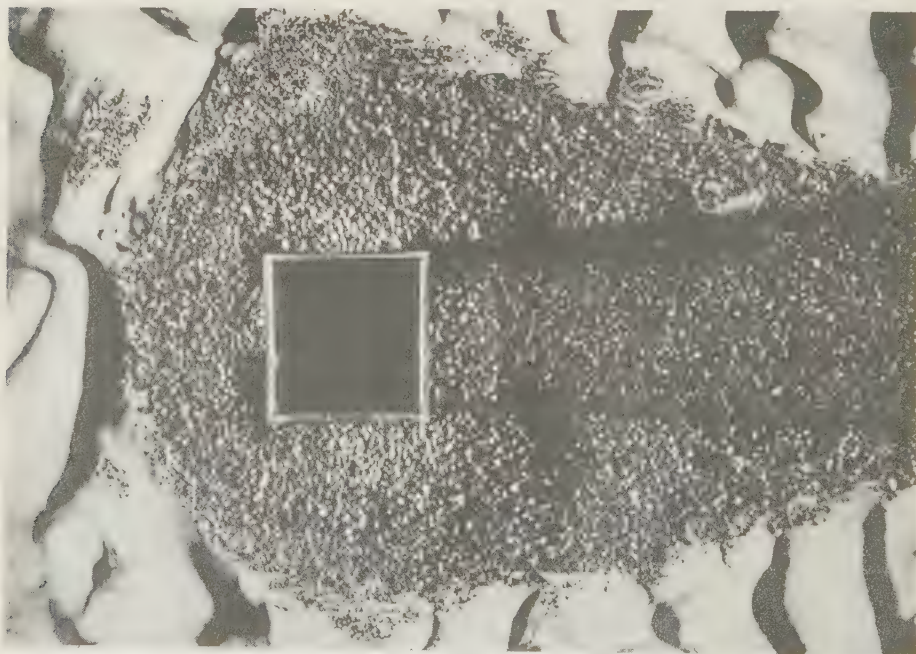


Fig. 6.59 Scour pattern after 4 hrs 4 - 8 mm

A similar series was run with the rectangular cylinder. The results were analogous, except that even for the 4 - 8 mm gravel, small scour holes developed at the upstream corners. The results are shown in Figs. 6.56 to 6.59.

6.6 Discussion of the experimental results.

Most of the conclusions of the theoretical analysis are supported by the experimental results. The secondary flow was found to be relatively weak compared to the primary flow. However, it was found strong enough to affect the primary flow in two important ways. The vertical secondary flow leads to a convection of vorticity towards the bed. This mechanism may lead to a considerable increase of boundary shear near the obstruction. Its effects are clearly seen from the unexpected shapes of the observed boundary shear distributions. The effect of their increase of boundary shear is very marked in the study of the development of scour around the circular cylinder. Similar scour patterns have been observed by Bonasoundas (1973) and at the Technical University of Norway (T. Carstens, personal communication).

A maybe more important effect of the secondary flow is that it reverses the flow in the lower fluid layers upstream of a bridge pier. The region near the pier is thereby effectively cut off from sediment supply. This effect may be one explanation of the fact that a rectangular pier causes more severe scour than a circular pier despite the fact that a circular pier causes larger boundary shear. At a blunter pier the region of reverse flow is larger than at a pier less blunt and therefore the sediment deficit becomes more pronounced. The scour hole is therefore enlarged until it reaches beyond the zone of reverse flow. Thereafter, bed load may slide down into the scour hole and reestablish the sediment equilibrium. This is probably the explanation to why a scour protection placed below general bed

level is more efficient than one placed at or above the bed level. If the protection is placed below bed level a scour hole is created, the edge of which may extend beyond the region of reversed flow. Therefore bed load is supplied to the scour hole from where it is transported into the wake region downstream of the cylinder by the flow in the scour hole. If the protection is placed at a sufficiently low level, this sediment transport rate balances the scouring action of the wake flow. If the scour protection is placed flush with the bed, the wake region is effectively cut off from sediment supply and severe scour may take place downstream of the scour protection.

The flow pattern was found to depend on the pier Reynolds' number but only for values of Re far lower than those which may be encountered for prototype conditions. For moderate values of Re and for high values of H/k_s and B/k_s the flow pattern was found to be independent of the mean velocity of the approach flow and to be very weakly, if at all, affected by the pier size, just as predicted by the theoretical analysis. The decrease of relative scour depth with increasing pier size which has been observed by several researchers is probably to be ascribed to the effects on the horizontal pressure gradients. Independent of the pier size maximum scour occurs for an approximately constant value of u_{*c} . As the flow pattern is practically independent of the pier size, the pressure gradients become inversely proportional to the pier width.

As predicted by theory nothing has been found but pressure forces to explain the scour under pipelines.

7 CONCLUSIONS

7.1 Scour around pipelines

The studies have shown that the initial undermining of a pipeline is certainly caused by pressure forces. As an initial scour hole is formed, shear stresses of the flow through the hole will extend it laterally and vertically. Thus, to protect a pipeline from undermining it should be sufficient to stabilize the bed near the pipe by means of a reversed filter. The filter may be designed according to usual filter criteria. The maximum pressure gradient in the subsoil is determined by the length of pipe periphery in contact with the bed and by the maximum momentaneous pressure difference between the upstream and downstream side of the pipe. According to this study, the latter is of order ρU^2 , where U is the mean velocity in the section above the crest of the pipe. All known experience indicates that if the filter is extended to the midheight of the pipe, the bed is safely protected.

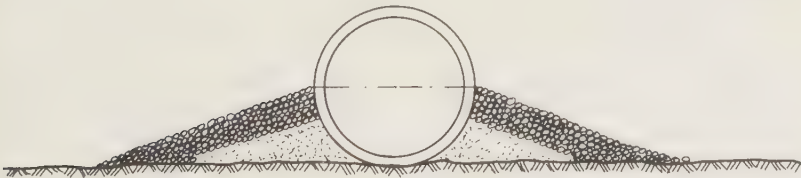


Fig. 7.1 Recommended scour protection for pipelines

The size of stones of the top layer of the filter may be determined by any critical shear stress formula using the shear stress of the approach flow and with due consideration to the inclination of the filter surface.

7.2 Scour around bridge piers

This study has shown that there are three important points to consider in the design of scour protection for bridge piers.

- (1) The size of the vortex system created about the pier. Into the area covered by the vortex system practically no sediment is transported. Therefore this area is the minimum area which has to be protected.
- (2) The strength of the vortex system. This is a very important factor in determining the required resisting properties of the scour protection.
- (3) The pressure field created by the pier. Here the temporal as well as the spatial distributions are of interest since the spatial distribution gives the mean pressure gradients, and the temporal distribution their respective maximum and minimum values, and a characteristic time-scale.

The size and shape of the surface under the vortex system has been experimentally determined for two cases, one involving a circular cylinder and one involving a rectangular pier shape. The vortex system has been found to depend on the Reynolds number, based on the pier width, but only up to a certain value. The experiments performed on the shape of the vortex system are believed to be representative for field conditions, where Re always will be great enough to make the shape of the vortex constant over the possible range of Re .

Thus, for circular and rectangular piers, the shape of the surface under the vortex can be deduced from the results presented in this study. For other pier shapes, the shape of this surface can be estimated by means of the simple shear approximation technique presented. This only gives the upstream edge of the surface but the edges can be extrapolated by straight lines, meeting

6 - 8 pier widths downstream of the pier.

The shear stresses near circular cylinders have been found to have a mean value of about 12 times the mean shear stress of the approach flow. Due to turbulent fluctuations, momentaneous shear stresses near the cylinder may reach a maximum value of 2.5 times the mean shear.

A rectangular pier, turned 45° to give a pointed nose type pier, creates shear stresses similar to the stresses created by a circular pier.

Blunter pier types, as the rectangular pier, with one side facing the flow, creates less dangerous shear stresses. The maximum mean shear is about 4 times the mean shear of the oncoming flow, and the maximum momentaneous shear stress is about 2.5 times the mean boundary shear.

The boundary shear under the vortex depends on the velocity, or vorticity distribution of the approach flow and increases with increasing velocity gradient. Thus, for a given type of velocity distribution and a given depth of flow the strength of the vortex increases with the mean velocity of the flow.

For the normally occurring boundary layer profile and for a given water depth, the strength of the vortex is directly proportional to the boundary shear of the oncoming flow.

It seems that the strength of the vortex increases with an increase of water depth, when the water depth is less than some, as yet undeterminate, multiple of the equivalent boundary roughness. This multiple presumably is of order 100.

The pressure gradients generated in the bed have been found to reach considerable magnitudes in certain cases. It is absolutely urged that pressure forces are included in the stability analysis

for a scour protection around a bridge pier. It is also recommended that the scour protection should be of a pervious type, as this type is judged to be the most efficient when seepage forces are acting. The pressure distributions observed for the highest values of Re of this study, are judged to be valid also for naturally occurring values of Reynolds number.

For pier shapes other than those studied in this study, the properties of the associated pressure field may be estimated by comparison with pressure data from a corresponding two-dimensional flow. Such values are often given in hydraulic handbooks. It should be noted however, that the 'three-dimensional' Reynolds number corresponds to a 'two-dimensional' Re , which is 5 - 10 times as large.

8 EXAMPLE OF HOW TO USE THE RESULTS OF THIS REPORT TO DESIGN A SCOUR PROTECTION

As an example of how the results of this study may be used to design a scour protection, the following case is considered;

Assume, that a circular bridge pier of 1 m diameter is to be provided with a rip-rap scour protection. The mean bed slope is about 0.1 % for a substantial reach upstream and downstream of the bridge. The design flow has been estimated to have a mean velocity of 1 m/s and a depth of 2 m. For this simple case the boundary shear of the approach flow is given by the du Boys formula (2.1)

$$\tau_0 = \gamma HS \quad (1)$$

where

γ = specific weight of fluid

H = depth of flow

S = mean slope of energy gradient

Thus,

$$\tau_0 = 9\,810 \cdot 2 \cdot 0.0001 \approx 2 \cdot \text{N/m}^2 \quad (2)$$

From the shear stress observations of chapter 6.2.1. it is seen that the mean shear is amplified 12 times near the pier, that is the maximum shear stress is about 24 N/m^2 . When the turbulent fluctuations are taken into consideration, the maximum shear stress may be about 60 N/m^2 .

The mean value of the horizontal pressure gradient is given by the pressure at the stagnation point, which is $\frac{1}{2} \rho U^2$, and the pressure at the point of separation, $-\rho U^2$. This pressure variation occurs over a length equal to one quarter of the pier periphery.

According to the findings of chapter 6.3.1 the maximum pressure fluctuation near the stagnation point is about $\frac{1}{4} \rho U^2$, and the maximum fluctuation near the point of separation is about $\frac{1}{2} \rho U^2$. Thus, the fluctuating pressure difference may be taken to have a maximum of $\frac{3}{4} \rho U^2$, which in this case corresponds to 750 N/m^2 .

According to chapter 6.4.2 the period T may be considered determined by the condition that the Strouhal number, Sh , should be about 0.2. Thus,

$$0.2 = \frac{B}{T \cdot U} = \frac{1}{T \cdot 2} ; \quad T = 2.5 \text{ sec.} \quad (3)$$

Now, to determine the seepage forces Sleath's formula (2.33) may be used. This formula is derived for waveinduced pressures and therefore the horizontal pressure gradient has to be transformed to an equivalent wavelength and the time scale has to be expressed as a wavenumber.

The mean pressure has its maximum at the stagnation point and its minimum at the separation point. If the distance between these points is taken as half a wavelength, the equivalent wavelength becomes πa , where a is the radius of the cylinder.

The wavelength of the fluctuating pressure is difficult to determine, but here it can be assumed to be approximately of the same order as that of the mean pressure. The wavenumber is zero for the mean pressure and $2\pi/T = 2\pi/2.5$ for the fluctuating component.

Thus, the pressure distribution at the boundary is given by:

$$p = p_1 \cos(-bx) + p_2 \cos\left(\frac{2\pi}{2.5} t - bx\right) \quad (4)$$

where

$$\left. \begin{aligned} p_1 &= 3 \cdot \frac{1}{2} \rho U^2 = 1500 \text{ N/m}^2 \\ p_2 &= 1.5 \cdot \frac{1}{2} \rho U^2 = 750 \text{ N/m}^2 \\ b &= 2\pi/(\pi a) = 2/a \end{aligned} \right\} \quad (5)$$

Now, by Sleaths formula (2.33)

$$p = \frac{\cosh b \left(\frac{k_x}{k_y} \right)^{1/2} (y + D)}{\cosh b \left(\frac{k_x}{k_y} \right)^{1/2} D} \left[p_1 \cos(-bx) + p_2 \cos\left(\frac{2\pi}{2.5} t - bx\right) \right] \quad (6)$$

Where

- k_x, k_y = the Darcy seepage coefficients for the x and y directions respectively
- D = the depth at which the vertical seepage velocity is zero
- y = the vertical coordinate with the value zero at the bed and with its positive direction upwards.

Assuming the bed to be isotropic, that is $k_x = k_y$, the pressure gradient at the bed is given by

$$\frac{\partial p}{\partial y} = \frac{2/a \cdot \sinh 2/a(y + D)}{\cosh 2/a D} \left[1500 \cos\left(-\frac{2x}{a}\right) + 750 \cos\left(\frac{2\pi}{2.5} t - \frac{2x}{a}\right) \right] \quad (7)$$

Thus, the maximum pressure gradient is

$$\begin{aligned} \left(\frac{\partial p}{\partial y} \right)_{\max} &= \frac{2}{a} \frac{\left[\sinh \frac{2y}{a} \cosh \frac{2D}{a} + \sinh \frac{2D}{a} \cosh \frac{2y}{a} \right]}{\cosh \frac{2D}{a}} \cdot 2250 \frac{\text{N}}{\text{m}^2 \cdot \text{m}} \approx \\ &\approx 2250 \cdot \frac{2}{a} \left[\sinh \frac{2y}{a} + \cosh \frac{2y}{a} \right] = \frac{4500}{a} \cdot e^{\frac{2y}{a}} \text{ N/m}^2 \cdot \text{m} \quad (8) \end{aligned}$$

for $D \gg a$

Thus, near the surface

$$\left(\frac{\partial p}{\partial y} \right)_{\max} \approx \frac{4500}{0.5} = 9000 \text{ N/m}^2 \cdot \text{m} \quad (9)$$

Assuming the filter material to have a γ_s of 2.65, a typical grain volume of V_s , an angle of repose $\rho = 38^\circ$ and a porosity $n = 0.4$ Martin and Aral's formula (2.44) gives, with $c = 1$,

$$\tan 38^\circ = \frac{V_s \cdot (26500 - 10000) \cdot \sin \alpha}{V_s \cdot (26500 - 10000) \cos \alpha - 9000 \cdot V_s (1 + 0.4)} \quad (10)$$

$$\alpha = 10^\circ$$

With this apparent angle of repose for instance White's formula (2.23) can be used to determine the necessary size of the rip-rap.

$$\tau_c = \frac{\pi r}{6} (\rho_s - \rho) g d \tan \alpha \quad (11)$$

$$24 = \frac{\pi r}{6} (2650 - 1000) \cdot 9.81 \cdot d \cdot 0.176 \quad (12)$$

According to White's experiments the value of r can be taken as 0.15 for turbulent flow

Thus

$$d = \frac{24 \cdot 6}{3.14 \cdot 0.15 \cdot 16500 \cdot 0.176} = 0.105 \text{ m} \quad (13)$$

Thus, the required size of the rip-rap is about 105 mm. The minimum area to be protected can be deduced from Fig. 6.51, scaling in direct proportion to the pier width.

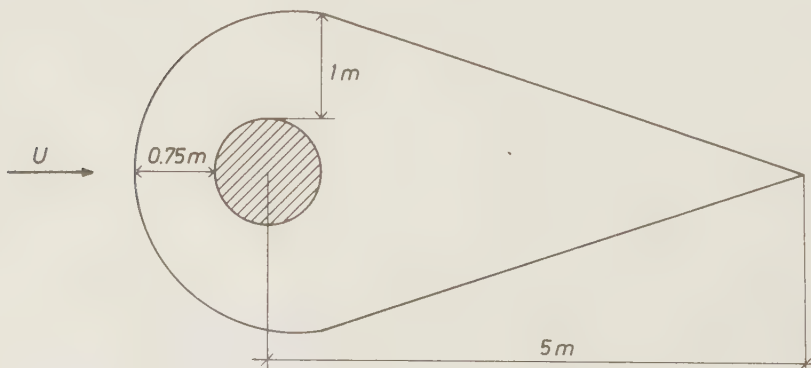


Fig. 8.1 Minimum area to be protected from scour

To prevent the bed sand from penetrating the rip-rap layer, this has to be provided with one or several filter layers. These filter layers may be designed according to for instance the recommendations of the U.S. Waterways Experiment station (1948) or by any preferred filter criteria. The vertical position of the rip-rap layer is of first importance. If the protective layer is placed at too high a level, it may well create secondary flow which leads to undermining of the filter. It is therefore recommended that the top of the rip-rap layer is placed at the lowest level to which the general bed is expected to scour.

If for the original bed $d_{15} = 0.1$ mm, $d_{50} = 0.3$ mm, $d_{85} = 0.7$ mm and if the filter layers are supposed to be fairly uniform then the grain size of the first filter layer is given by $d < 5 d_{85} = 3.5$ mm, $4 d_{15} = 0.4$ mm $< d < 20 d_{15} = 2$ mm, $d < 2^5 d_{50} = 7.5$ mm. Thus, the first layer should have a grain size of about 2 mm. The size of the successive layers can be taken as 5 times the grain size of the preceding layer i.e. 10, 50 and 250 mm respectively. As this gives an unnecessarily coarse filter, the layer could be taken as 1.5, 6, 25 and 110 mm respectively. The minimum required thickness of a specific layer is about 5 times the grain size of the layer.

LIST OF REFERENCES

- ACHENBACH, E., 1968 "Distribution of local pressure and skin friction around a circular cylinder in cross-flow up to $Re = 5 \times 10^6$ ". J. Fluid Mech. vol. 34 part 4, pp. 625 - 639.
- BONASOUNDAS, M., 1973 "Strömungsvorgang und Kolkproblem am runden Brückenpfeiler". Versuchsanstalt für Wasserbau der Technischen Universität München - Oskar v. Miller Institut - Bericht Nr 28.
- BREUSEFS, H.N.C. 1972 "Local scour near offshore structures" Delft Hydraulics Laboratory. Publication No. 105.
- BROWN, C.B., 1950 "Sediment transportation" in H. Rouse (ed.) "Engineering Hydraulics". John Wiley and Sons, New York.
- CARSTENS, M.R., 1966 "Similarity laws for localized scour" Journal of the Hydraulics Division, ASCE, Vol. 92 No. HY3, pp. 13 - 36.
- CHABERT, J., ENGELDINGER, P.
"Étude des affouillements autour des piles de ponts". Laboratoire National D'Hydraulique, Chatou, Série A, Octobre 1956.
- CHITALE, S.V., 1962 Discussion of "Scour at bridge crossings" by E. M. Laursen. Trans. ASCE, Vol. 127, pp. 191 - 196.
- COLBY, B.R., 1964 "Discharge of sands and mean velocity relationships in sand-bed streams". U.S. Geological Survey, Prof. Paper 462-A, Washington, D.C.

- DARWIN, C.G., 1953 "Note on hydrodynamics".Proc. Camb. Phil. Soc. 49, pp. 342 - 54.
- DU BOYS, P., 1879 Ann. d. Ponts et Chauss., (5) 18 p 149
- EGIAZAROFF, J.W. 1967 General report on river hydraulics. 12 th Congress of the IAHR, Proc, Vol.5, p 409.
- EINSTEIN, H.A., 1942 "Formulas for the transportation of bed load".Trans. ASCE, Vol. 107, pp. 561 - 573.
- EINSTEIN, H.A., 1950 "The bedload function for sediment transportation in open channels".U.S. Department of Agriculture, Soil Conservation Service, Technical Bulletin 1026, Washington D.C.
- EINSTEIN, H.A., BARBAROSSA, N., 1952 "River channel roughness".Trans. ASCE, Vol. 117, pp. 1121 - 1146.
- EINSTEIN, H.A., EL SAMNI, E-S.A., 1949 "Hydrodynamic forces on a rough wall". Review of Modern Physics, American Institute of Physics, Lancaster, Pa., Vol.21, No.3, pp. 520 - 24
- ENGELS, H., 1894 "Schutz der Strompfeilerfundamente gegen Unterspülung". Zeitschrift für Bauwesen, pp. 408 - 16.
- ENGELUND, F., 1966 "Hydraulic resistance of alluvial streams". Journal of the Hydraulics Division, ASCE, Vol. 92, No.HY2. pp. 315 - 326.
- ENGELUND, F., HANSEN, E., 1967 "A monograph on sediment transport in alluvial streams". Teknisk Forlag, Copenhagen.

- GERRARD, J.H., 1966 J. Fluid Mech. Vol. 25, p 401.
- GRASS, A. J., 1970 "Initial instability of fine bed sand".
Journal of the Hydraulics Division, ASCE,
Vol. 96, No. HY3, pp. 619 - 32.
- HANCU, S., 1971 "Sur le calcul des affouillements locaux
dans la zone des piles du pont". 14th
Congress of the IAHR, Proceedings, Vol.3.
- HAWTHORNE, W.R., 1954 "Secondary flow about struts and airfoils"
Journal of the Aeronautical Sciences,
Vol.21, No.1, pp. 588 - 608.
- HJORTH, P., 1972 "Lokal erosion och erosionsverkan vid av-
loppsledning i kustnära områden". Insti-
tutionen för vattenbyggnad, Tekniska Hög-
skolan i Lund, Bulletin Serie B nr 21.
- HJORTH, P., 1974 "Lokal erosion kring bropelare och rörled-
ningar". Institutionen för vattenbyggnad,
Tekniska Högskolan i Lund, Bulletin Serie
A nr.30.
- HYDRAULICS RESEARCH STATION WALLINGFORD, 1973
" A study of scour around submarine pipe-
lines". Hydraulics Research Station, Walling-
ford, Berkshire, Report No. INT 113.
- INGLIS, C.C., 1949 "The behaviour and control of rivers and
canals". Central Water Power Irrigation
and Navigation Report, Poona Research
Station, Research Publication 13, part
I and II.
- INGLIS, C.C., THOMAS, A.R., JOGLEKAR, D.V., 1939
"The protection of bridge piers against scour".
Annual Report of Work Done During 1938-39,
Central Irrigation and Hydrodynamics Re-
search Station, Poona. Research Publication

- INGLIS, C.C., THOMAS, A.R., JOGLEKAR, D.V., 1941
 "Annual Report of Work Done 1940-41".
 Central Irrigation and Hydrodynamic Research Station, Poona, Research Publication No.5.
- INGLIS, C.C., THOMAS, A.R., JOGLEKAR, D.V., 1942
 India Central Irrigation and Hydrodynamic Research Station, Poona, Research Publication No.5. pp. 35 - 38.
- KALINSKE, A.A., 1947
 "Movement of sediment as bed load in rivers".
 Trans. Am. Geo. Union, Vol. 28, August 1947.
- KEUTNER, C., 1932
 "Stromungsvorgänge an Strompfeilern von verschiedenen Grundrissformen und ihre Einwirkung auf die Flusssohle". Die Bautechnik, Vol.10, No.12.
- KJEDSEN, S.P., GJØRSVIK, O., BRINGAKER, K.G., 1974
 "Experiments with local scour around submarine pipelines in a uniform current".
 River and Harbour Laboratory at the Technical University of Norway, Oppdrag No.600849.
- KLINGEMAN, P.C., 1973
 "Hydrologic evaluations in bridge pier scour design". Journal of the Hydraulics Division, ASCE, Vol.99, No. HY12, pp. 2175-2184.
- LAMB, H., 1932.
 "Hydrodynamics". Cambridge at the University press, Sixth edition.
- LARRAS, J., 1963
 "Profondeurs maximales d'érosion des fonds mobiles autour des piles en rivière". Annales des ponts et chaussées, Vol. 133, No. 4, pp. 411 - 24.

- LAURSEN, E.M., 1951 "Progress report of model studies of scour around bridge piers and abutments". Highway Research Board, Research Report No. 13-B.
- LAURSEN, E.M., 1952 "Observations on the nature of scour". State University of Iowa, Iowa City, Proc. of the 5 th Hydraulic Conference.
- LAURSEN, E.M., 1955 "Model-prototype comparisons of bridge-pier scour". Highway Research Board, Proc. Vol.34, pp. 138 - 193.
- LAURSEN, E.M., 1958 " The total sediment load of streams". Journal of the Hydraulics Division, ASCE, Vol.84, No. HY1.
- LAURSEN, E.M., 1960 "Scour at bridge crossings". Journal of the Hydraulics Division, ASCE, Vol.86, No. HY2.
- LAURSEN, E.M., 1962 "Scour at bridge crossings". Trans. ASCE, Vol.127, part 1, paper No. 3294.
- LAURSEN, E.M., 1963 "A generalized model study of scour around bridge piers and abutments". Proc. Minnesota International Hydraulics Convention.
- LAURSEN, E.M., 1970 "Bridge design considering scour and risk". Transportation Engineering Journal of ASCE, Vol.96, No. TE2 pp. 149 - 64.
- LAURSEN, E.M., HUBBARD, P.G., 1954 "Model-prototype comparison and bridge pier scour". Proc. County Engineers Conference, Iowa State College of Agriculture and Mechanical Arts, Ames, Iowa.

LAURSEN, E.M., TOCH, A., 1951

"Model studies of scour around bridge piers and abutments". Second Progress Report, State University of Iowa Reprints in Engineering. Reprint No. 107, Reprinted from Highway Research Board Proceedings, December, 1961.

LAURSEN, E.M., TOCH, A., 1953

" A generalized model study of scour around bridge piers and abutments". Proceedings of the 5th general meeting of the IAHR, Minneapolis, Paper No.157.

LAURSEN, E.M., TOCH, A., 1956

"Scour around bridge piers and abutments". Iowa Highway Research Board, Bulletin No. 4.

LIGHTHILL, M.J., 1956 a "Drift". J. Fluid. Mech., Vol. 1, pp. 31-53.

LIGHTHILL, M.J., 1956 "The image system of a vortex in a rigid sphere". Proc. Camb. Phil. Soc., Vol. 52, No. 2, pp. 317 - 21.

LIGHTHILL, M.J., 1957 "Corrigenda to drift". J. Fluid Mech. Vol. 2, pp. 311 -21

LIU, P. L-F., 1973 "Damping of water waves over porous bed". Journal of the Hydraulics Division, ASCE, Vol. 99, No. HY12, pp. 2263 - 72.

MARTIN, C.S., ARAL, M.M., 1971

"Seepage forces on interfacial bed particles". Journal of the Hydraulics Division, ASCE, Vol. 97, No. HY7, pp. 1081 - 1100.

MEYER-PETER, E., MULLER, R., 1948

"Formulas for bed-load transport". Proc.
2nd general meeting of the IAHR, Stock-
holm, Paper No. 52.

MILNE- THOMSON, L.M., 1962

"Theoretical Hydrodynamics". Fourth edi-
tion, MacMillan and Co, London.

MOORE, W.L., MASCH, F.D., 1963

"Influence of secondary flow on local scour
at obstructions in a channel". U.S. De-
partment of Agriculture, Miscellaneous
Publication 970, Paper No. 36 .

NICOLLET, G., RAMETTE, M., 1971

"Affouillements au voisinage de piles
de pont cylindriques circulaires".
14th Congress of the IAHR, Proceedings,
Vol.3, Paper C-38.

O'BRIEN, M.P., MORISON, J. R., 1952

"The forces exerted by waves on objects".
Am. Geophysical Union, Trans, Vol. 33, No. 1.

O'BRIEN, M.P., RINDLAUB, B.D., 1934

"The transportation of bed load by stream".
National Research Council, Transactions
of the American Geophysical Union, 15th
annual meeting, Part II, pp. 593 - 603.

POSEY, C.J., 1949

"Why bridges fail in floods". Civil
Engineering, Vol 19, pp. 42 - 90.

POSEY, C.J., 1963

"Scour at bridge piers". Civil Engineering,
May 1963, pp. 48 - 49.

- POSEY, C.J., 1974 "Tests of scour protection for bridge piers". Journal of the Hydraulics Division, ASCE, Vol.100, No. HY12, pp. 1773 - 1783.
- POSEY, C.J., SYBERT, J.H., 1961 "Erosion Protection of production structures". General Assembly of the IAHR, Proceedings, pp. 1157 - 1162.
- RAWN, A.M., BOWERMAN, F.R., 1950 "Factors influencing and limiting the location of sewer ocean outfalls". First Conference on Coastal Engineering, Proceedings, Long Beach, California.
- ROBERSON, J.A., LIN, C.Y., RUTHERFORD, G.S., STINE, M.D., 1971 "Turbulence effects on drag of sharp-edged bodies". Journal of the Hydraulics Division, ASCE, Vol.98, No. HY7, pp. 1127 - 1203.
- ROMITA, P.L., 1960 Discussion of "Scour at bridge crossings". Journal of the Hydraulics Division, ASCE, Vol.86, No. HY9, pp. 151 - 152.
- ROSENHEAD, (ed.), (1963) "Laminar Boundary Layers". Oxford University Press.
- ROSHKO, A., 1954 "On the development of turbulent wakes from vortex streets". NACA, Report 1191.
- SHIELDS, A., 1936 "Anwendung der Ähnlichkeits-Mechanik und der Turbulenzforschung auf die Geshiebewegung". Preussische Versuchsanstalt für Wasserbau und Schiffbau, Berlin, Mitteilungen, Heft 26.

- SCHOKLITSCH, A., 1914 "Über Schleppkraft und Geschiebepbewegung". Engelmann, Leipzig.
- SCHWARTZ, K., 1929 "Comparative experiments on the influence of the size of particles of a river bottom on the depth of excavation occurring in the vicinity of bridge piers". Hydraulic Laboratory Practice, edited by J.R. Freeman, P. 201.
- SCHWIND, R.G., 1962 "The three-dimensional boundary layer near a strut". Gas Turbine Laboratory, Massachusetts Institute of Technology, Report No. 67.
- SHAW, T.L., STARR, M.R., 1972 "Shear flow past circular cylinder". Journal of the Hydraulics Division, ASCE, Vol.98, No. HY3, pp. 461 - 471.
- SHEN, H.W., SCHNEIDER, V.R., KARAKI, S.S., 1966 "Mechanics of local scour". Engineering Research Centre, Colorado State University, Report, No. CER 66 HWS 22. SUPPLEMENT: "Methods of reducing scour". Report, No. CER 66 HWS 36.
- SHEN, H.W., SCHNEIDER, V.R., KARAKI, S., 1969 "Local scour around bridge piers". Journal of the Hydraulics Division, ASCE, Vol.95 No. HY6, pp. 1919 - 40.
- SLEATH, F.A., 1970 "Wave induced pressures in beds of sand". Journal of the Hydraulics Division, ASCE, Vol.96, No. HY2, pp. 367 - 78.

SON, J.S., HANRATTY, T.J., 1969

"Velocity gradients at the wall for flow around a cylinder at Reynolds numbers from 5×10^3 to 10^5 ". J. Fluid Mech., Vol.35, part 2, pp. 353 - 368.

STEWART, R.W., 1939

"Safe foundation depths for bridges to protect from scour". Civil Engineering, Vol.9, No. 6, pp. 336 - 337.

TANAKA, S., YANO, M., 1967

"Local scour around a circular cylinder". 12th Congress of the IAHR, Proceedings, Vol. 3 Fort Collins.

THOMAS, Z., 1967

"An interesting hydraulic effect occurring at local scour". 12 th Congress of the IAHR, Proc. Vol. 3, Fort Collins.

TIMONOFF, V.E., 1929

"Experiments on the spacing of bridge piers in the case of parallel bridges". Hydraulic Laboratory Practice, edited by J.R. Freeman, Am. Soc. of Mech. Engrs., New York.

TISON, L.J., 1937

"Affouillement autour des piles de ponts en rivière". Academie Royale de Belgique, Bullentin de la Classe des Sciences, 15e Série XXIII.

TISON, L.J., 1939

"L'érosion et le transport de matériaux solides dans les cours d'eau". Reports and papers for the postponed meeting of the Int. Assoc. for Hydraulics Structures Research, Stockholm, Paper No. 3, pp. 71 - 90.

TISON, L.J., 1940

"Erosion autour de piles de ponts en rivière". Annales des Travaux Publics de Belgique, Vol. 41, No. 6.

- TOFALLETI, F.B., 1969 "Definitive computations of sand discharge in rivers". Journal of the Hydraulics Division, ASCE, Vol. 95, No. HY1, pp. 225 - 48.
- TOOMRE, A., 1959 "The viscous secondary flow ahead of an infinite cylinder in a uniform parallel shear flow". J. Fluid Mech, Vol. 7 No. 1, pp. 145 - 155.
- TOWNSEND, D.R., FARLEY, D.W., 1973 "Design criteria for submarine pipeline crossings". Journal of the Hydraulics Division, ASCE, Vol. 99, No. HY10.
- U.S. WATERWAYS EXPERIMENT STATION, 1948 "Laboratory investigation of filters for Enid and Granada Dams". U.S. Waterways Experiment Station, Technical Memorandum No. 30245.
- VANONI, V.A., BROOKS, N.H., KENNEDY, J.F., 1961 "Lecture notes on sediment transportation and channel stability". California Institute of Technology, Report No. KH-R-1.
- VANONI, V.,(CHAIRMAN), 1966 "Sediment transportation mechanics: Initiation of motion". Progress report of the Task Committee on Preparation of Sedimentation Manual, Journal of the Hydraulics Division, ASCE, Vol. 92, No. HY2, pp. 291 - 314.

VANONI, V., (CHAIRMAN), 1971

"Sediment transportation mechanics: H. Sediment discharge formulas". By the Task Committee for Preparation of Sediment Manual, Committee on Sedimentation of the Hydraulics Division, Journal of the Hydraulics Division, ASCE, Vol. 97, No. HY4, pp. 523 - 567.

VENKATADRI, C., RAO, C.M., HUSSAIN. S.T., ASTHANA, K.C., 1965

"Scour around bridge piers and abutments". Irrigation and Power, Vol. 22, No. 1, pp. 35 - 42.

WATTERS, G.Z., RAO, V.P., 1971

"Hydrodynamic effects of seepage on bed particles". Journal of the Hydraulics Division, ASCE, Vol. 97, No. HY3, pp. 421 - 39.

WHITE, C.M., 1940

"The equilibrium of grains on the bed of a stream". Royal Society, Proceedings, A 174.

YARNELL, D.L., NAGLER, F.A., 1931

"A report upon a hydraulic investigation of North Carolina standard reinforced concrete bridge pier number P-401-R and modifications thereof". Iowa Institute of Hydraulic Research, University of Iowa, Iowa City.

